A STUDY ON BEARING CAPACITY OF SUBMERGED ROCK FOUNDATION

A Thesis submitted in partial fulfillment of the requirements
for the award of the Degree of

Master of Technology

in

Geotechnical Engineering

Civil Engineering Department

by

NIRUPAM BARUAH

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DEPARTMENT OF CIVIL ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY

ROURKELA - 769008

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CERTIFICATE

This is to certify that the thesis entitled "**A study on Bearing Capacity of Submerged Rock Foundation**" submitted by **Mr. Nirupam Baruah** (Roll No. 212CE1021) in partial fulfilment of the requirements for the award of Master of Technology Degree in Civil Engineering with specialization in Geo-Technical Engineering at National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

Date: **Dr. N. Roy**

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ABSTRACT

For more urbanization, engineering structures are not limited to underground tunnels or metro stations, engineers are constructing underground multistoried building even underground city. Therefore accurate assessment of rock strength is necessary for the rational design of underground structures. Bearing capacity is one of the most important factor for assessment of rock strength. Various studies on determination of bearing capacity for intact as well as jointed rock mass have been performed. Concisely, various studies have been performed for determination of bearing capacity of rock mass under dry condition while for submerged condition, it is outnumbered. In this study, determination of submerged bearing capacity of rock mass is attempted. Most cases, the rock substratum is considered to be homogeneous, intact, but the practical scenario does not recommend so. In-situ rock mass variability renders the deterministic analysis to be inefficient and hence there arises the necessity for probabilistic or reliability analyses to model the uncertainties. Rather than calculating a deterministic factor of safety, a reliability based analysis is more appropriate for geotechnical design. Present study comprises of different aspects regarding ultimate bearing capacity of rock foundations for both dry and submerged condition. These are, development of algorithmic modelling for determination of ultimate bearing capacity for both dry and submerged condition, analysis of behavior of ultimate bearing capacity due to the effect of submergence, comparative study between theorem mechanisms and reliability analysis through Monte-Carlo simulation for various parameters of rock mass. The data generated from the mentioned aspects are represented in graphical form and histogram bar charts are developed from Monte Carlo simulation.

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NOMENCLATURE

α : Orientation angle.

β : Reliability Index.

γ : Total unit weight of the rock mass.

γ_{sat} : Saturated unit weight of rock mass.

γ_w : Unit weight of water.

γ' : Effective unit weight.

θ : Angle of velocity discontinuity line (CD) with horizontal.

μ_M : Mean values of M (Margin of Safety).

μ_Q : Mean values of Q (Load).

μ_R : Mean values of R (Resistance).

ξ_n : Function of α, θ, ϕ_i & ϕ_j.

ρ_{RQ} : Correlation Coefficient between R and Q (Load).

σ_F : Standard Deviations of F (Factor of Safety).

σ_M : Standard Deviations of M (Margin of Safety).

σ_Q : Standard Deviations of Q (Load).

σ_R : Standard Deviations of R (Resistance).

σ_M^2 : Variances of M (Margin of Safety).

σ_Q^2 : Variances of Q (Load).

σ_R^2 : Variances of R (Resistance).

ϕ_i : Angle of friction of intact rock mass.

φ_j : Angle of friction of jointed rock mass.

B : Footing Width.

c_i : Cohesion of intact rock mass.

c_j : Cohesion of jointed rock mass.

CI_{95} : A 95% confidence interval.

CI_{99} : A 99% confidence interval.

d_w : Depth of water from foundation base.

$E[F]$: Expected values of F.

F : Factor of Safety.

f_n : Function of ξ & ϕ_j for two sided mechanism.

g_n : Function of ξ & ϕ_j for one sided mechanism.

h_C : Height from ground level to point C in mechanism.

h_D : Height from ground level to point D in mechanism.

LCL_{95}: Lower confidence limit at 95% confidence

M : Margin of Safety.

n_o : Number of joint set.

N_{ci} : Bearing Capacity Coefficient.

N_{cj} : Bearing Capacity Coefficient.

N_q : Bearing Capacity Coefficient.

N_γ : Bearing Capacity Coefficient.

N_γ^{sub} : Bearing Capacity Coefficient.

$N\gamma_w$: Additional factor depends on groundwater depth.

P_f : Probability of Failure.

q : Surcharge.

q_u : Ultimate bearing capacity for dry condition.

q_{uw} : Ultimate bearing capacity for submerged condition.

Q : Load.

R : Resistance.

S_i : Spacing of i^{th} joint set.

s_x : The standard error.

t_{95} : The t value for a two-tailed test, alpha level of .05 with a given degrees of freedom.

t_{99} : The t value for a two-tailed test, alpha level of .01 with a given degrees of freedom.

UCL_{95}: Upper confidence limit at 95% confidence.

$\overline{X}$: The sample mean.

x_n : n^{th} Co-ordinate Point.

X_n : n^{th} Variable.

MATLAB VARIABLES

alpha : Orientation angle (α).

B : Footing Width (B).

ci : Cohesion of Intact Rock (c_i).

cj : Cohesion of Jointed Rock (c_j).

dw : Depth of water from foundation base (d_w).

f1 : Function of ξ & ϕ_j for two sided mechanism (f_n).

f2 : Function of ξ & ϕ_j for two sided mechanism (f_n).

f3 : Function of ξ & ϕ_j for two sided mechanism (f_n).

f4 : Function of ξ & ϕ_j for two sided mechanism (f_n).

g1 : Function of ξ & ϕ_j for one sided mechanism (g_n).

g2 : Function of ξ & ϕ_j for one sided mechanism (g_n).

g3 : Function of ξ & ϕ_j for one sided mechanism (g_n).

g4 : Function of ξ & ϕ_j for one sided mechanism (g_n).

GWC : Ground water condition, either ‘DRY’ or ‘SUBMERGED’.

hC : Height from ground level to point C in mechanism.

hD : Height from ground level to point D in mechanism.

mechanism : Mechanism to be used either one sided ‘OS’ or two sided ‘TS’.

Nci : Bearing capacity coefficient (N_{ci}).

Ncj : Bearing capacity coefficient (N_{cj}).

Nq : Bearing capacity coefficient (N_q).

Nr : Bearing capacity coefficient (N_γ).

Nrsub: Bearing Capacity Coefficient (N_γ^{sub}).

Nrw : Additional factor depends on groundwater depth ($N\gamma_w$).

phii : Angle of friction of Intact Rock (ϕ_i).

phij : Angle of friction of Jointed Rock (ϕ_j).

q : Surcharge (q).

qu : Ultimate bearing capacity for both dry and submerged condition (q_u & q_{uw}).

r : Bulk unit weight (γ).

reff : Effective unit weight (γ').

rsat : Saturated unit weight (γ_{sat}).

rw : Unit weight of water (γ_w).

theta : Angle of velocity discontinuity line (CD) with horizontal (θ).

xi1 : Function of α, θ, ϕ_i & ϕ_j (ξ_n).

xi2 : Function of α, θ, ϕ_i & ϕ_j (ξ_n).

xi3 : Function of α, θ, ϕ_i & ϕ_j (ξ_n).

xi4 : Function of α, θ, ϕ_i & ϕ_j (ξ_n).

xi5 : Function of α, θ, ϕ_i & ϕ_j (ξ_n).

xi6 : Function of α, θ, ϕ_i & ϕ_j (ξ_n).

CHAPTER 1

INTRODUCTION

1. INTRODUCTION

1.1 ORIGIN OF PROJECT

Except from some soft-rock, most rock masses are excellent or in other words, hard enough for foundation material. However, for very huge construction with unusually high loads and special requirements like 100-storey skyscrapers, large bridge, arch-dam, nuclear power plants, underground city and others, it is very essential to study the strength of rock mass. Therefore, in those cases, it is essential to justify and determine the ultimate bearing capacity, and also the allowable load for shallow foundations. In other cases, when the loads are large and the rock mass is weak or highly fragmented, a more rigorous method to establish the ultimate bearing capacity of the rocks is needed.

Sutcliffe et al. (2004), Yang and Yin (2005), Merifield et al. (2006), and Saada et al. (2008) have performed various studies on rock foundations. In these studies, effect of groundwater and joint spacing on the bearing capacity had not been investigated. Thereafter, the upper bound theorem of limit analysis was used by Imani et al. (2012) to investigate the bearing capacity of submerged jointed rock foundations.

Imani et al. (2012) performed splendid studies on upper bound solution for the bearing capacity of submerged jointed rock foundations. These studies include various relationships between the different bearing capacity parameters such as relationship between ultimate bearing capacity for dry rock and ultimate bearing capacity for submerged rock for different ground water depth, width of foundations along with cohesion and frictional angle of both intact and jointed rock. He suggested that upper bound theorem should be used for relatively simple structures.

1.2 PRESENT STUDY COMPONENTS

Since Imani et al. (2012) is the first published document for determining the bearing capacity for submerged rock by upper bound solution, further studies should be carried out. Most cases, the rock substratum is considered to be homogeneous, intact, but the practical scenario does not recommend so. In-situ rock mass variability renders the deterministic analysis to be inefficient and hence there arises the necessity for probabilistic or reliability analyses to model the uncertainties. Rather than calculating a deterministic factor of safety, a reliability based analysis is more appropriate for geotechnical design. Present study comprises of the following:

Firstly, An Algorithmic modelling has been developed as a step by step procedure for calculation of bearing capacity. Mentioned algorithmic program is developed with the help of MATLAB language. The program consists of computational commands in a sequence and the computational commands are created in MATLAB script files. When the script file is executed, computational commands in the order as they are enlisted are also executed and by putting different input parameters e.g. footing width, orientation angle, rock mass strength properties, physical properties, water table location and surcharge, output parameter i.e. ultimate bearing capacity of rock mass can be evaluated.

Secondly, a study on behavior of ultimate bearing capacity due to the effect of submergence is attempted. Graphical representation is carried out to observe variation of ultimate bearing capacity with reduction of water table.

Thirdly, a comparative study between theorem mechanisms of upper bound theorem for determination of submerged bearing capacity of rock mass has been investigated. Graphical representation is carried out to observe the mentioned comparative study.

At last, a reliability based study through Monte-Carlo simulation is attempted. A stochastic model with random input parameters of various strength parameters and different ground water locations is created in Microsoft Excel Spreadsheet to identify the performance of the correlations of upper bound theorem for determination of submerged bearing capacity of rock mass. The data generated from the simulation is represented as histogram bar charts. Different statistical descriptors like mean, median, standard deviation and many others are generated for ten thousand samples. Furthermore, probability distribution function and confidence interval were also evaluated.

This study will help researcher to investigate issues related to bearing capacity of rock mass with submerged condition.

1.3 LOWER BOUND THEOREM

The lower bound theorem states that the collapse load obtained from any statically admissible stress field will underestimate the true collapse load. A statically admissible stress field is one which firstly satisfies the equations of equilibrium, secondly satisfies the stress boundary conditions and finally does not violate the yield criterion. Two basic assumptions must be made about the material in order to apply the lower bound theorem.

1. The material is assumed to be perfectly plastic, that is, the material exhibits ideal plasticity with an associated flow rule without strain hardening or softening. The level of ductility exhibited by a jointed rock mass is dependent on the imposed stress state.
2. It is assumed that all deformations or changes in the geometry of the rock mass at the limit load are small and therefore negligible.

It is recognized that the lower bound theorem has been applied less frequently than the upper bound theorem as it is easier to construct a kinematically admissible failure mechanism than it is to construct a statically admissible stress field. Furthermore, the lower bound theorem is often difficult to apply to problems involving complex loading and geometry, particularly if it is necessary to construct the stress fields manually.

1.4 UPPER BOUND THEOREM

The upper bound theorem states that the rate of work done by actual forces is less than or equal to the rate of energy dissipation in any kinematically admissible velocity fields. A kinematically admissible velocity field is compatible with the velocities at the boundary of the geometrical masses. The solution obtained from upper bound theorem, in the non-conservative side, is not less than the actual solution by a kinematically admissible velocity field. To obtain the better solution (lower upper bounds), work has to be done for as many trial kinematically admissible velocity fields as possible. The lowest possible upper bound solution is sought with an optimization scheme by trying various possible kinematically admissible failure mechanisms.

The upper bound theorem is a powerful tool for stability analysis and has been widely used in many areas of geotechnical design. However, for practical problems which involve nonhomogeneous material, complicated loadings and complex geometries, this theorem quite difficult to apply. An upper bound to the collapse loads may obtain by postulating a collapse mechanism, and computing the ratio of the plastic dissipation associated with this mechanism to the work done by the applied loads.

CHAPTER 2

LITERATURE REVIEW

2. LITERATURE REVIEW

Metropolis and Ulam (1949) stated the ‘Monte Carlo method’ with several statements mentioned below:

1. Monte Carlo method is a statistical approach to the study of differential equations, or more generally, of integro-differential equations that occur in various branches of the natural sciences.
2. The mathematical description of Monte Carlo method is the study of a flow which consists of a mixture of deterministic and stochastic processes.

Chen and Drucker (1969) stated that slip-line solutions give neither rigorous lower bound nor rigorous upper bound solutions on the actual collapse load.

Sutcliffe et al. (2004) concluded the followings:

1. Inclusion of single week joint, reduction in strength is significantly affected by both the strength of the joint relative to the properties of the intact rock material and to the orientation of the joint set.
2. Inclusion of two joint sets, the strength of the joints as well as the joint orientation significantly affects the result, although in this case the angle between the two joint sets also plays an important role.
3. The inclusion of a third joint set vertically oriented results in a further loss in ultimate bearing capacity of up to 40% as compared to the results for a rock mass with two joint sets only. Parameters similar to those in the two joint case were again found to be critical.

Yang and Yin (2005) established a theorem on the ultimate bearing capacity of a strip footing with the modified failure criterion using the generalized tangential technique. Assumptions were made such as the strip footing is long enough for consideration of plain strain problem, rigid triangular blocks of symmetrical translation failure mechanism, homogenous and isotropic rock masses are idealized as a perfectly plastic material, and follow the associated flow rule and lastly, the failure of the rock masses is governed by a modified HB failure criterion. Using the technique, an MC linear failure criterion, which is tangent to the actual HB failure criterion, is proposed to calculate the rate of external work and internal energy dissipation. Suggested equations in this theorem executes with a maximum difference of less than 0.5% for bearing capacity factors. It is found that the surcharge load and self-weight have effects on the ultimate bearing capacity, and that the contribution related to uniaxial compressive strength can be separated from the ultimate bearing capacity while the contributions related to surcharge load and unit weight cannot be separated from the ultimate bearing capacity. It should be noted that lower bound solution is less than or equal to actual solution.

Merifield et al. (2006) investigated bearing capacity of a surface strip footing resting on a rock mass whose strength can be described by the generalized Hoek–Brown failure criterion and drawn the following conclusions:

1. The effect of ignoring rock weight can lead to a very conservative estimate of the ultimate bearing capacity. This is particularly the case for poorer quality rock types.
2. Estimating the ultimate bearing capacity of a rock mass using equivalent Mohr–Coulomb parameters was found to significantly overestimate the bearing capacity.
3. Existing numerical solutions for weightless rock masses are generally conservative.

Imani et al. (2012) concluded that application of the upper bound theorem was used for relatively simple configurations because of the non-homogeneity and discontinuous nature of rock masses. Further observation stated that the Spacing Ratio, different mechanical properties of the intact rock and the joint sets did not show a remarkable effect on the bearing capacities obtained from the upper bound solution. Also it has been observed that the shape (i.e. either a straight line or a log-spiral curve) of velocity discontinuity lines does not have any significant effect on the submerged bearing capacity. Furthermore another conclusion is that the bearing capacity of dry and submerged rocks indicate that submergence of the rock below the footing base reduces the contribution of the rock weight in the bearing capacity.

CHAPTER 3

THEORY

3. THEORY

3.1 INTRODUCTION

From civil engineering aspects, foundation is commonly known as the lowest part of a structure that transmits its weight to the underlying soil or rock. Foundations are generally classified into two major categories – shallow foundation and deep foundation. Considering splendid research studies of foundations, it can be concluded that if the depth of foundation is less than or equal to 4 times the width of foundation then the foundation is shallow foundation or otherwise if the depth of foundation is more than 4 times the width of foundation then the foundation is deep foundation.

Most commonly bearing capacity is the load-carrying capacity of foundation soil or rock which enables it to bear and transmit loads from a structure. Whereas ultimate bearing capacity is the theoretical maximum pressure which a foundation can support without the occurrence of shear failure. In other words, ultimate bearing capacity is the external load applied per unit area of the foundation after which shear failure occurs.

Intact rock permeability is small and negligible in comparison to the permeability of joints. However, in field, the whole rock mass below the water table is saturated after a specific time lag and leads to buoyancy of the rock mass. For various combinations of mechanical properties of intact rock and joint set, there is a depth beyond which the groundwater has no effect on the bearing capacity. This depth is called the 'critical depth'.

3.2 DETERIMINATION OF BEARING CAPACITY

3.2.1 ASSUMPTIONS

For the calculations of bearing capacity of shallow foundations on rocks for both dry and submerged conditions following assumptions were made:

1. A rock mass containing two orthogonal tight joint sets was considered.
2. The concept of spacing ratio proposed by Serrano and Olalla (1996) is used to account the joint spacing.
3. The mechanical properties of the intact rock and the joint sets are not affected by the groundwater.
4. The Mohr-Coulomb failure criteria was used for both the intact rock and the joint sets.

5. Orientation angles (α) equal to 15°, 30° and 45° were considered for one of the joint sets.
6. A two-sided symmetrical failure mechanism is used for the case of α=45° and a one-sided asymmetrical failure mechanism is used for α=15° and 30°.
7. Velocity discontinuity line at failure plane is considered to be a straight line and its beginning and end points are located at the junction of the junction of the two joint sets, not within intact rock block.

3.2.2 FAILURE MECHANISM

The mechanism by which the minimum bearing capacity can be determined is formally known as the most appropriate failure mechanism. Imani et al. (2012) suggested two different failure mechanism depending upon joint set condition –

1. Two-sided mechanism.
2. One-sided mechanism.

3.2.2.1 Two-sided Mechanism

Two-sided mechanism is symmetrical in shape. This mechanism is considered only with centric and vertical footing in the foundations. A hodograph diagram of two sided failure mechanism is shown below.

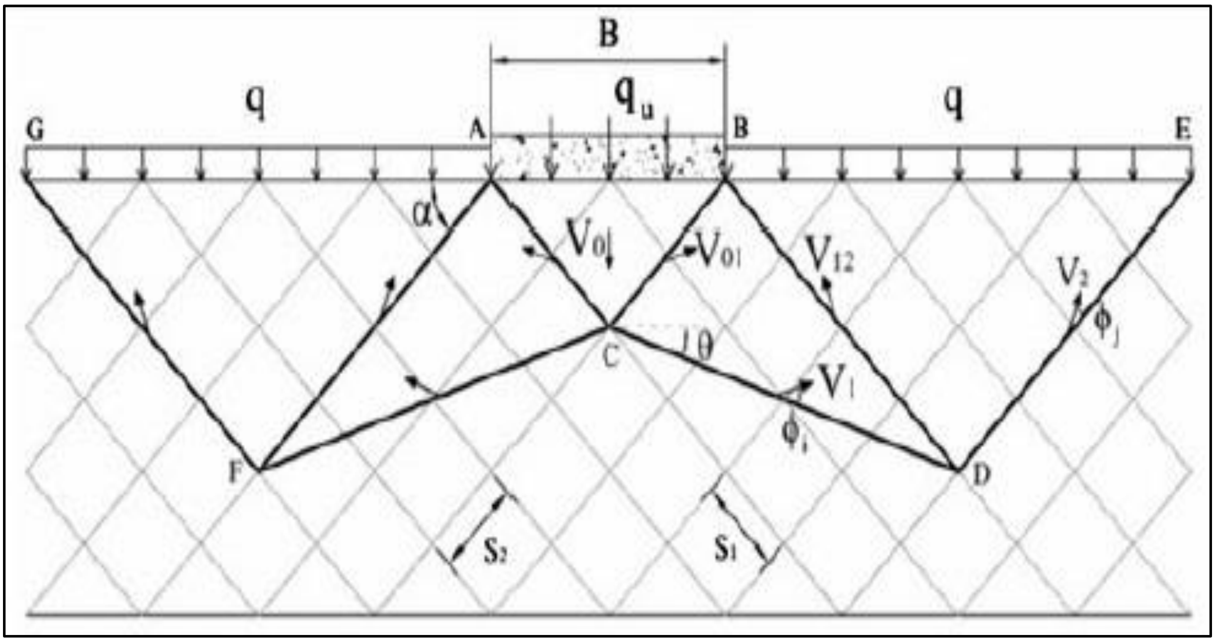


Fig. 3.1: Hodograph diagram of two sided mechanism

3.2.2.2 One-sided Mechanism

One-sided mechanism is asymmetrical in shape. In presence of joint set, the failure mechanism may be affected by the joint set and shape is converted to asymmetrical. A hodograph diagram of one sided failure mechanism is shown below.

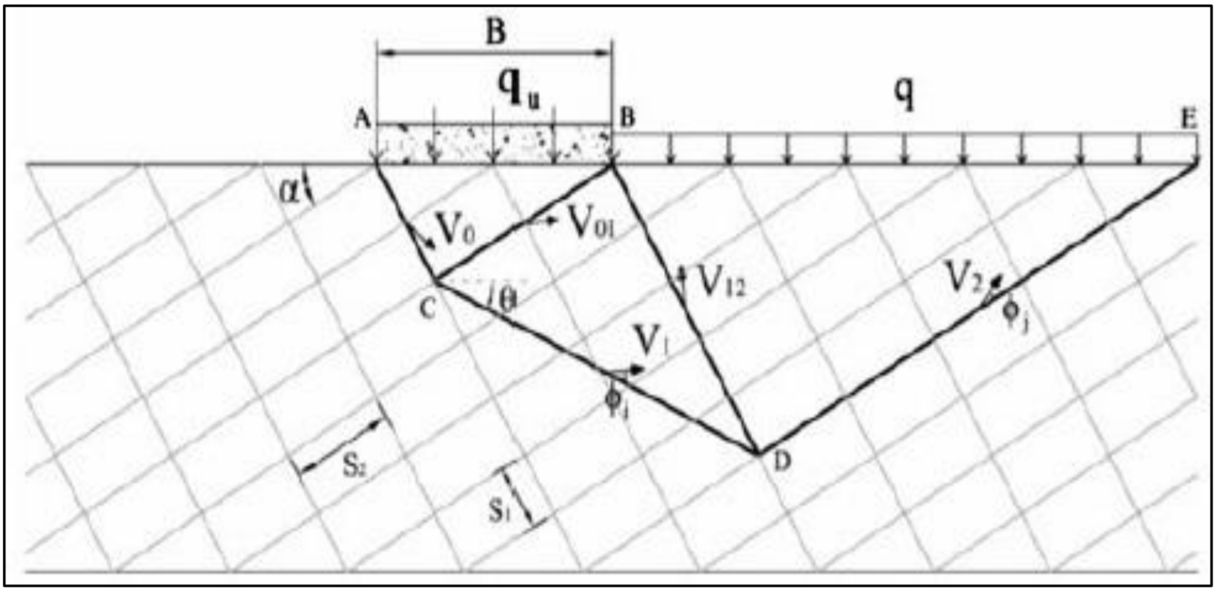


Fig. 3.2: Hodograph diagram of one sided mechanism

3.2.3 CALCULATIONS

Two different sets of equation were proposed for each of the mechanisms. Each set contains different equations for dry condition as well as submerged condition for determining submerged bearing capacity of rock mass. For ease of calculations, a rock mass containing orthogonal tight joint sets were considered. By knowing footing width (B), orientation angle (α), rock mass strength properties (c_i, c_j, ϕ_i, ϕ_j), physical properties (γ, γ_{sat}), water table location (d_w), surcharge (q) one can find out the ultimate bearing capacity of rock mass by the following equations:

For dry condition: $q_u = c_i N_{ci} + c_j N_{cj} + q N_q + 0.5 \gamma B N_\gamma$ … eq. 1

For submerged condition: $q_u = c_i N_{ci} + c_j N_{cj} + q N_q + 0.5 \gamma B N_\gamma^{sub}$ ………eq. 2

The angle of velocity displacement line (CD) with the horizontal direction is given by:

$$\theta = \tan^{-1}\left(\frac{n_0 S_1}{B \cos\alpha}\right) - \alpha \qquad \text{.... eq. 3}$$

Assumptions made for calculations of ultimate bearing capacity are as follows –

$\xi_1 = \alpha + \theta - \phi_i - \phi_j$ ....eq. 4

$\xi_2 = \alpha + \theta - \phi_i + \phi_j$ ....eq. 5

$\xi_3 = \alpha + \phi_j$ ....eq. 6

$\xi_4 = \alpha + \theta$ ....eq. 7

$\xi_5 = \phi_i - \theta$ ....eq. 8

$\xi_6 = \alpha - \phi_j$ ....eq. 9

Now putting f_n is the function of ξ & ϕ_j for two sided mechanism -

$f_1 = \sin\xi_6 - \sin\xi_5 \cos\xi_1$ ....eq. 10

$f_2 = -\sin\xi_6 \sin\xi_5 + \cos\xi_1$ ....eq. 11

$f_3 = \sin 2\phi_j \cos\xi_2 + \sin\xi_1$ ....eq. 12

$f_4 = \sin 2\phi_j + \cos\xi_2 \sin\xi_1$ ....eq. 13

Again putting g_n as the function of ξ & ϕ_j for one sided mechanism -

$g_1 = \cos\xi_1 \sin\xi_2 - \sin 2\phi_j$ ....eq. 14

$g_2 = \cos\xi_1 - \sin\xi_2 \sin 2\phi_j$ ....eq. 15

$g_3 = \sin 2\phi_j \cos\xi_2 + \sin\xi_1$ ....eq. 16

$g_4 = \sin 2\phi_j + \cos\xi_2 \sin\xi_1$ ....eq. 17

Bearing capacity coefficients can be find out from these following equations –

<u>For two-sided mechanism</u>

$$N_{cj} = \frac{2\cos\phi_j \cos\alpha}{f_1}\left[\cos^2\xi_5 + \frac{f_2}{f_3} \text{ x } \tan\xi_4 \left(\sin^2\xi_2 + \frac{f_4}{\tan\alpha}\right)\right] \quad \text{.........eq. 18}$$

$$N_{ci} = \frac{f_2}{f_1} \text{ x } \frac{2\cos\phi_i \cos\alpha}{\cos\xi_4} \quad \text{..eq. 19}$$

$$N_q = \frac{f_2 f_4}{f_1 f_3} \text{ x } \frac{2\sin\xi_3 \tan\xi_4}{\tan\alpha} \quad \text{..eq. 20}$$

$$N_\gamma = \cos\alpha \left[\frac{2 f_2}{f_1} \text{ x } \cos\alpha \tan\xi_4 \text{ x } \left(\sin\xi_5 + \frac{f_4}{f_3} \text{ x } \frac{\sin\xi_3 \tan\xi_4}{\tan\alpha}\right) - \sin\alpha\right] \quad \text{.........eq. 21}$$

For one-sided mechanism

$$N_{cj} = \frac{\cos\phi_j \cos\alpha}{\cos\xi_3} \left[\tan\alpha + \frac{\cos^2\xi_2}{g_1} + \frac{g_2}{g_1 g_3} \times \tan\xi_4 \left(\sin^2\xi_2 + \frac{g_4}{\tan\alpha} \right)\right] \text{.........eq. 22}$$

$$N_{ci} = \frac{g_2}{g_1} \times \frac{\cos\phi_i \cos\alpha}{\cos\xi_3 \cos\xi_4} \quad \text{...eq. 23}$$

$$N_q = \frac{g_2 g_4}{g_1 g_3} \times \frac{\tan\xi_3 \tan\xi_4}{\tan\alpha} \quad \text{...eq. 24}$$

$$N_\gamma = \cos\alpha \left[\frac{g_2}{g_1} \times \frac{\cos\alpha \tan\xi_4}{\cos\xi_3} \times \left(\sin\xi_5 + \frac{g_4}{g_3} \times \frac{\sin\xi_3 \tan\xi_4}{\tan\alpha} \right) - \sin\alpha \right] \text{.........eq. 25}$$

Submerged bearing capacity coefficient (N_γ^{sub}) for both the mechanisms can be find out from the following equation:

$$N_\gamma^{sub} = \frac{\gamma'}{\gamma} N_\gamma + \frac{d_w}{B} \left(1 - \frac{\gamma'}{\gamma} \right) N_{\gamma w} \quad \text{...eq. 26}$$

It had been considered that the water table is located at a depth d_w from the foundation base. Location of the water table can vary to any depth of rock mass. Therefore, two different heights had been established to distinguish two different boundaries for the water table and an additional factor ($N_{\gamma w}$) had been established that depends on the groundwater depth. Referring to fig. 3.1 and fig.3.2 water table location varies from foundation base level to initial point (point C) of velocity discontinuity line CD (h_C) for one condition while for another varies up to last point (point D) of velocity discontinuity line CD (h_D). It has been observed that beyond the point D, water table has no effect on bearing capacity and ultimate bearing capacity will be same as in the dry condition. These two boundary heights can be determined by the following equations -

$$h_C = B \sin\alpha \cos\alpha \quad \text{..eq. 27}$$

$$h_D = B \cos^2\alpha \tan(\alpha + \theta) \quad \text{..eq. 28}$$

For different condition of water table depth with boundary heights additional factor ($N_{\gamma w}$) can be evaluated from the following equations:

For two sided mechanism

a) If $0 \leq d_w \leq h_C$

$$N_{\gamma w}= \frac{d_w}{B}\,\frac{2}{\sin 2\alpha}\left(1 + \frac{2\,f_2}{f_1}\sin\xi_5 - \frac{2\,f_2\,f_4}{f_1\,f_3}\sin\xi_3 \right) + \frac{4\,f_2\,f_4}{f_1\,f_3}\,\frac{\sin\xi_3\,\tan\xi_4}{\tan\alpha} - 2 \quad \ldots\text{eq. 29}$$

b) If $h_C \leq d_w \leq h_D$

$$N_{\gamma w} = \frac{4\,f_2}{f_1}\tan\xi_4\cos\alpha\left(\frac{\cos\xi_4\sin\xi_5}{\sin\theta} + \frac{f_4}{f_3}\,\frac{\sin\xi_3}{\sin\alpha}\right) - \frac{2\,d_w}{B}\,\frac{f_2}{f_1}\,\frac{1}{\cos\alpha}\left(\frac{\cos\xi_4\sin\xi_5}{\sin\theta} + \frac{f_4}{f_3}\,\frac{\sin\xi_3}{\sin\alpha}\right) + \frac{B}{d_w}\cos\alpha\left[\frac{2\,f_2}{f_1}\cos\alpha\tan\xi_4\sin\xi_5\left(1 - \frac{\cos\alpha\sin\xi_4}{\sin\theta}\right) - \sin\alpha\right]$$

……….. eq. 30

<u>For one sided mechanism</u>

a) if $0 \leq d_w \leq h_C$

$$N_{\gamma w}= \frac{d_w}{B}\,\frac{2}{\sin 2\alpha}\left(1 + \frac{g_2}{g_1}\,\frac{\sin\xi_5}{\cos\xi_3} - \frac{g_2\,g_4}{g_1\,g_3}\tan\xi_3 \right) + \frac{2\,g_2\,g_4}{g_1\,g_3}\,\frac{\tan\xi_3\,\tan\xi_4}{\tan\alpha} - 2 \quad \ldots\ldots\text{eq. 31}$$

b) if $h_C \leq d_w \leq h_D$

$$N_{\gamma w} = \frac{2\,g_2}{g_1}\,\frac{\sin\xi_4\cos\alpha}{\cos\xi_3}\left(\frac{\sin\xi_5}{\sin\theta} + \frac{g_4}{g_3}\,\frac{\sin\xi_3}{\sin\alpha\cos\xi_4}\right) - \frac{d_w}{B}\,\frac{g_2}{g_1}\,\frac{1}{\cos\alpha\cos\xi_3}\left(\frac{\cos\xi_4\sin\xi_5}{\sin\theta} + \frac{g_4}{g_3}\,\frac{\sin\xi_3}{\sin\alpha}\right) + \frac{B}{d_w}\cos\alpha\left[\frac{g_2}{g_1}\,\frac{\cos\alpha\tan\xi_4\sin\xi_5}{\cos\xi_3}\left(1 - \frac{\cos\alpha\sin\xi_4}{\sin\theta}\right) - \sin\alpha\right]$$

…… eq. 32

It should be noted that if $d_w > h_D$, groundwater has no effect on bearing capacity and therefore, N_γ^{sub} will be equal to N_γ for both mechanisms.

3.3 ALGORITHMIC ANALYSIS

An algorithm is an effective method expressed as a finite list of well-defined instructions for calculating a function. Starting from an initial state and initial input, the instructions describe a computation that, when executed, proceeds through a finite number of well-defined successive states, eventually producing output and terminating at a final ending state. Most commonly algorithm is known as a step by step procedure for calculations or solving a problem.

3.3.1 FLOWCHART

A flowchart is a type of diagram that represents an algorithm, workflow or process, showing the steps as boxes of various kinds, and their order by connecting them with arrows.

Commonly flowchart is a diagrammatic representation of a solution to any problem. Flowcharts are used in analyzing, designing, documenting or managing a process or program in various fields. There are many different types of flowcharts, and each type has its own repertoire of boxes and notational conventions.

3.3.2 MATLAB Programming

MATLAB is a high-level language and interactive environment for numerical computation, visualization, and programming developed by MathWorks Inc., Natick, Massachusetts, USA. MATLAB is a fourth generation programming language which is powerful and popular language for technical computing and most prominently easy to use. MATLAB can be used for mathematical computations, algorithm development, modeling, matrix manipulations, plotting of functions and data, creation of user interfaces, interfacing with programs written in other languages, including C, C++, Java, and FORTRAN and many others.

Most commonly, a program is a sequence of computational commands. MATLAB programs are generally written in MATLAB script file. A script file is a list of MATLAB commands, called a program that is saved in a file. When the script file is executed, MATLAB executes the commands in the order in which they are listed, and in which all the variables are defined within the script file. MATLAB provides several tools that can be used to control the flow of a program. Conditional statements and the switch structure (Appendix III) make it possible to skip commands or to execute specific groups of commands in different situations.

3.4 RELIABILITY ANALYSIS

3.4.1 IMPORTANCE

In-situ rock mass variability renders the deterministic analysis to be inefficient; and, hence there arises the necessity for probabilistic or reliability analyses to model the uncertainties. Rather than calculating a deterministic factor of safety, a reliability based analysis is more appropriate for geotechnical design. Such analysis indicates the performance and reliability of a geotechnical problem, and can be used for risk-based decision making.

3.4.2 DEFINITION

The reliability of an engineering system can be defined as the confidence on its ability to fulfill its design purpose for some time-period. The theory of probability provides the fundamental basis to measure this ability. The reliability of a structure can be viewed as the probability of its satisfactory performance, according to some performance functions, for a specific service and subjected to extreme conditions within a stated time-period. In estimating this probability, system uncertainties are modeled using random variables with mean values, variances, and probability distribution functions.

3.4.3 OVERVIEW

Generally, reliability analysis deals with the relation between the loads carried by a system and its ability to carry those loads. Both the loads and the resistance may be uncertain, so the result of their interaction is also uncertain. Loads and resistance may include not only forces and stresses, but also seepage, settlement, and any other phenomena that might become design considerations. The values of both load and resistance are uncertain, so these variables have mean or expected values, variances, and co-variances, as well as other statistical descriptors.

To initiate a reliability analysis, random fields of soil or rock mass properties are commonly generated to derive the required statistical parameters, e.g. mean and standard deviation. A method of reliability analysis is then selected for determining the probability of failure and the reliability index. Some commonly used techniques are the Monte Carlo simulation, First Order methods and Point Estimate method.

3.4.4 RELIABILITY INDEX

Reliability index is defined as the ratio of mean value of margin of safety to variance of margin of safety. Reliability index depends on correlation coefficient, as reliability index varies with the condition of correlation coefficient. Calculations of the reliability index are more difficult when it is expressed in terms of the factor of safety, because factor of safety is the ratio of two uncertain quantities while Margin of safety is their difference.

The margin of safety, M, is the difference between the resistance and the load:

Mathematically - $$M = R - Q \qquad \text{eq. 33}$$

From the elementary definitions of mean and variance, it follows that regardless of the probability distributions of Resistance and Load, the mean value of Margin of Safety is

$$\mu_M = \mu_R - \mu_Q \qquad \text{eq. 34}$$

and the variance of Margin of Safety is

$$\sigma_M^2 = \sigma_R^2 + \sigma_Q^2 - 2\rho_{RQ}\sigma_R\sigma_Q \qquad \text{eq. 35}$$

Then, a reliability index, β, is defined as

$$\beta = \frac{\mu_M}{\sigma_M} \qquad \text{eq. 36}$$

Therefore,

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2 - 2\,\rho_{RQ}\,\sigma_R\,\sigma_Q}} \qquad \text{eq. 37}$$

Which expresses the distance of the mean margin of safety from its critical value (M = 0) in units of standard deviation. If the load and resistance are uncorrelated, the correlation coefficient is zero, and

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \qquad \text{eq. 38}$$

eq. 37 and eq. 38 shows variation of reliability index with the condition of correlation coefficient.

Factor of safety, F, is defined as

$$F = \frac{R}{Q} \qquad \text{eq. 39}$$

Failure occurs when F=1, and a reliability index is defined by

$$\beta = \frac{E\,[\,F\,]-1}{\sigma_F} \qquad \text{eq. 40}$$

For very small values of reliability index the probability of failure is actually slightly larger for the Normal distribution than for the others.

3.4.5 STEPS IN RELIABILITY ANALYSIS

To explain what is involved in the various approaches to reliability calculations, it is useful to set out the steps explicitly. The goal of the analysis is to estimate the probability of failure, with the understanding that 'failure' may involve any unacceptable performance. The steps are:

i. **Establish an analytical mode**: There must be some way to compute the margin of safety, factor of safety, or other measure of performance. It can be simple or it can be an elaborate computational procedure. There may be error, uncertainty, or bias in the analytical model, which can be accounted for in the reliability analysis.

ii. **Estimate statistical descriptions of the parameters**: The parameters include not only the properties of the geotechnical materials but also the loads and geometry. Usually, the parameters are described by their means, variances, and co-variances, but other information such as spatial correlation parameters or skewness may be included as well.

iii. **Calculate statistical moments of the performance function**: Usually this means calculating the mean and variance of the performance function.

iv. **Calculate the reliability index**: Often this involves the simple application of reliability index equations. Sometimes, as in the Hasofer–Lind method, the computational procedure combines this step with the previous one.

v. **Compute the probability of failure**: If the performance function has a well-defined probabilistic description, such as the Normal distribution, this is a simple calculation. In many cases the distribution is not known or the intersection of the performance function with the probabilistic description of the parameters is not simple. In these cases the calculation of the probability of failure is likely to involve further approximations.

3.4.6 ERROR PROPAGATION

The evaluation of the uncertainty in the computed value of the margin of safety and the resulting reliability index is a special case of error propagation. Basically, uncertainties in the values of parameters propagate through the rest of the calculation and affect the final result. Generally, each of the computational steps involves some error and uncertainty on its own, and uncertainties in the original measurements of object properties will affect the numbers calculated at each subsequent step.

3.4.7 SOLUTION TECHNIQUES

In more typical cases, the analyst must usually employ a technique that yields an approximation to the true value of the reliability index and the probability of failure. Several methods are available, each having advantages and disadvantages.

3.4.7.1 The First Order Second Moment (FOSM) Method

This method uses the first terms of a Taylor series expansion of the performance function to estimate the expected value and variance of the performance function. It is called a second moment method because the variance is a form of the second moment and is the highest order statistical result used in the analysis. If the number of uncertain variables is N, this method requires either evaluating N partial derivatives of the performance function or performing a numerical approximation using evaluations at 2N+1 points.

3.4.7.2 The Second Order Second Moment (SOSM) Method

This technique uses the terms in the Taylor series up to the second order. The computational difficult is greater, and the improvement in accuracy is not always worth the extra computational effort. The Second Order Second Moment methods have not found wide use in geotechnical applications.

3.4.7.3 The Point Estimate Method

Rosenblueth (1975) proposed a simple and elegant method of obtaining the moments of the performance function by evaluating the performance function at a set of specifically chosen discrete points. One of the disadvantages of the original method is that it requires that the performance function be evaluated 2^N times, and this can become a very large number when the number of uncertain parameters is large. Recent modifications reduce the number of evaluations to the order of 2N, but introduce their own complications.

3.4.7.4 The Hasofer–Lind Method

Hasofer and Lind (1974) proposed an improvement on the First Order Second Moment method based on a geometric interpretation of the reliability index as a measure of the distance in dimensionless space between the peak of the multivariate distribution of the uncertain parameters and a function defining the failure condition. This method usually requires iteration in addition to the evaluations at 2N points. The acronym First Order Reliability method often refers to this method in particular.

3.4.7.5 Monte Carlo Method

Essentially, Monte Carlo method is a statistical approach to the study of differential equations, or more generally, of integro-differential equations that occur in various branches of the natural sciences. In this approach the analyst creates a large number of sets of randomly generated values for the uncertain parameters and computes the performance function for each set. The statistics of the resulting set of values of the function can be computed and Reliability Index is calculated directly. The method has the advantage of conceptual simplicity, but it can require a large set of values of the performance function to obtain adequate accuracy.

Computational operations can be divided roughly into two classes. Firstly, production of random values with their frequency distribution equal to those which govern the change of each parameter. Secondly, calculation of the values of those parameters which are deterministic, i.e. obtained algebraically from the others. Monte Carlo simulation is categorized as a sampling method because the inputs are randomly generated from probability distributions. The data generated from the simulation can be represented as histograms or converted to error bars, reliability predictions, tolerance zones, and confidence intervals.

One interesting feature of the method is that it allows one to obtain the values of certain given operations on functions obeying a differential equations, without point to point knowledge of the functions which are solutions of the equations. One more advantage is that we avoid dealing with multiple integrations or multiplications of the probability matrices, but instead sample single chains of events.

3.4.8 RECOMMENDATION

Before carrying out a reliability analysis it is very important to understand that most practical methods of reliability analysis involve approximations, even if one or more steps are exact. It is very obvious that different methods will give different answers. Therefore, it is often a good idea to compare results from two or more approaches to gain an appreciation of the errors involved in the computational procedures.

3.4.9 STATISTICAL TERMS

Statistical terms used in reliability analysis only are described below:

3.4.9.1 Population

The group from which data are collected or a sample is selected. The population encompasses the entire group for which the data are alleged to apply.

3.4.9.2 Sample

An individual or group, selected from a population, from whom or which data are collected.

3.4.9.3 Mean

The mean is simply the arithmetic average of a distribution of scores and it provides a single, simple number that gives a rough summary of the distribution. It is important to remember that although the mean provides a useful piece of information, it does not tell anything about how spread out the scores are (i.e., variance) or how many scores in the distribution are close to the mean.

3.4.9.4 Median

The median is the score in the distribution that marks the 50^{th} percentile. That is, 50% of the scores in the distribution fall above the median and 50% fall below it. Median is often used when distribution scores are dived into two equal groups (a median split). The median is also a useful statistic to examine when the scores in a distribution are skewed or when there are a few extreme scores at the high end or the low end of the distribution.

3.4.9.5 Mode

The mode is the least used of the measures of central tendency because it provides the least amount of information. The mode simply indicates which score in the distribution occurs most often, or has the highest frequency.

3.4.9.6 Range

The range is simply the difference between the largest score (the maximum value) and the smallest score (the minimum value) of a distribution.

3.4.9.7 Interquartile Range

Unlike the range, the interquartile range is the difference between the score that marks the 75th percentile (the third quartile) and the score that marks the 25th percentile (the first quartile). If the scores in a distribution were arranged in order from largest to smallest and then divided into groups of equal size, the interquartile range would contain the scores in the two middle quartiles.

3.4.9.8 Variance

The variance provides a statistical average of the amount of dispersion in a distribution of scores. In general, variance is used more as a step in the calculation of other statistics (e.g., analysis of variance) than as a stand-alone statistic.

3.4.9.9 Standard Deviation

The word ‘deviation’, in this case, refers to the difference between an individual score in a distribution and the average score for the distribution. So if the average score for a distribution is 10, and an individual score is 12, then the deviation is 2. The other word 'standard' means typical, or average. So a standard deviation is the typical, or average, deviation between individual scores in a distribution and the mean for the distribution.

3.4.9.10 Normal Distribution

The normal distribution is known in statistics as a theoretical distribution. There are a number of statistics that begin with the assumption that scores are normally distributed. When this assumption is violated (i.e., when the scores in a distribution are not normally distributed), there can be dire consequences.

A more familiar name for the normal distribution is the 'bell curve', because a normal distribution forms the shape of a bell. Fig. 3.3, represented as a normal distribution curve, which has three fundamental characteristics. First, it is symmetrical, meaning that the upper half and the lower half of the distribution are mirror images of each other. Second, the mean, median, and mode are all in the same place, in the center of the distribution (i.e., the top of the bell curve). Because of this second feature, the normal distribution is highest in the middle. Finally, the normal distribution is asymptotic, meaning that the upper and lower tails of the distribution never actually touch the baseline, also known as the x-axis.

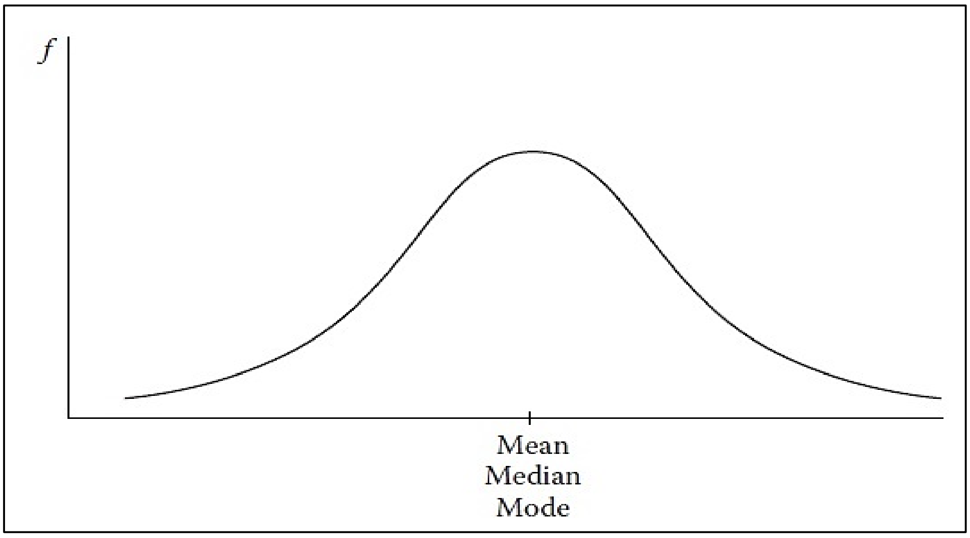


Fig. 3.3: Line diagram representing standard shape of normal distribution.

3.4.9.11 Skew

When a sample of scores is not normally distributed (i.e., not the bell shape), there are a variety of shapes it can assume. If there are a few scores creating an elongated tail at the higher end of the distribution, it is said to be positively skewed. If the tail is pulled out toward the lower end of the distribution, the shape is called negatively skewed. So a positively skewed distribution will have a higher mean than median, and a negatively skewed distribution will have a smaller mean than median.

3.4.9.12 Kurtosis

Kurtosis refers to the shape of the distribution in terms of height, or flatness. When a distribution has a peak that is higher than that found in a normal, bell-shaped distribution, it is called ‘leptokurtic’. When a distribution is flatter than a normal distribution, it is called ‘platykurtic’. A leptokurtic distribution will have a greater percentage of scores closer to the mean and fewer in the upper and lower tails of the distribution, whereas a platykurtic distribution will have more scores at the ends and fewer in the middle than will a normal distribution.

3.4.9.13 Standard Error

Technically, a standard error is, in effect, the standard deviation of the sampling distribution of some statistic (e.g., the mean, the difference between two means, the correlation coefficient, and many others.). Or in other words, standard error is the denominator in the formulas used to calculate many inferential statistics. This is because the standard error is the measure of how much random variation we would expect from samples of equal size drawn from the same population.

3.4.10 MICROSOFT EXCEL SPREADSHEET

Microsoft Excel spreadsheet is a spreadsheet application developed by Microsoft, Albuquerque, New Mexico, U.S. It features calculation, graphing tools, pivot tables, and a macro programming language called Visual Basic for Applications. Microsoft Excel has the basic features of all spreadsheets using a grid of cells arranged in numbered rows and letter-named columns to organize data manipulations like arithmetic operations. It is very useful for statistical, engineering and financial needs. In addition, it can display data as line graphs, histograms and charts, and with a very limited three-dimensional graphical display.

CHAPTER 4

ANALYSIS & DISCUSSION

4. ANALYSIS AND DISCUSSION

4.1 ALGORITHMIC ANALYSIS

4.1.1 FLOWCHART

A flowchart or in a broad sense a diagrammatic representation of the complete algorithm for determination of bearing capacity has been developed. The flowchart represents workflow or process of the algorithm in terms of different boxes in order and by connecting them with arrows. With the help of this flowchart one can understand the complete process of algorithm i.e. from input (footing width, orientation angle, rock mass strength properties, physical properties, water table location, and surcharge) to output (ultimate bearing capacity) and how the mechanisms are selected or how the water table condition is selected. Fig. 4.1 represents the flowchart of the algorithmic model for determination of ultimate bearing capacity of rock foundation for both dry and submerged condition. It can be easily noticed that after division of mechanism condition, workflow for each mechanism is quite same, but for each one, theorem conditions are different and mathematical equations are also different. The symbols used in the flowchart are described in the Appendix I.

4.1.2 ALGORITHMIC MODEL

For determination of ultimate bearing capacity of rock mass for both dry and submerged condition, an algorithmic model has been developed in MATLAB script. Commands and programs implemented in this model are mentioned in Appendix II and Appendix III respectively. For any case, with known footing width (B), orientation angle (α), rock mass strength properties (c_i, c_j, ϕ_i, ϕ_j), physical properties (γ, γ_{sat}), water table location (d_w), surcharge (q), ultimate bearing capacity (q_u) of rock mass for both dry and submerged condition can be determined with the help of this model in MATLAB software.

It is necessary to mention that some command lines are not short enough to write in a single line. Therefore the incomplete portion is written in the next line or if it is not completed in that line also then another line is added and so on. Line numbers are addressed before each line to understand more precisely. But in MATLAB script file, line numbers will be addressed automatically and if each single command line is converted into many lines without ellipsis (…) then the program will run with error. So it is better to write a single command line in one line as a line in MATLAB script file may contain 4096 characters.

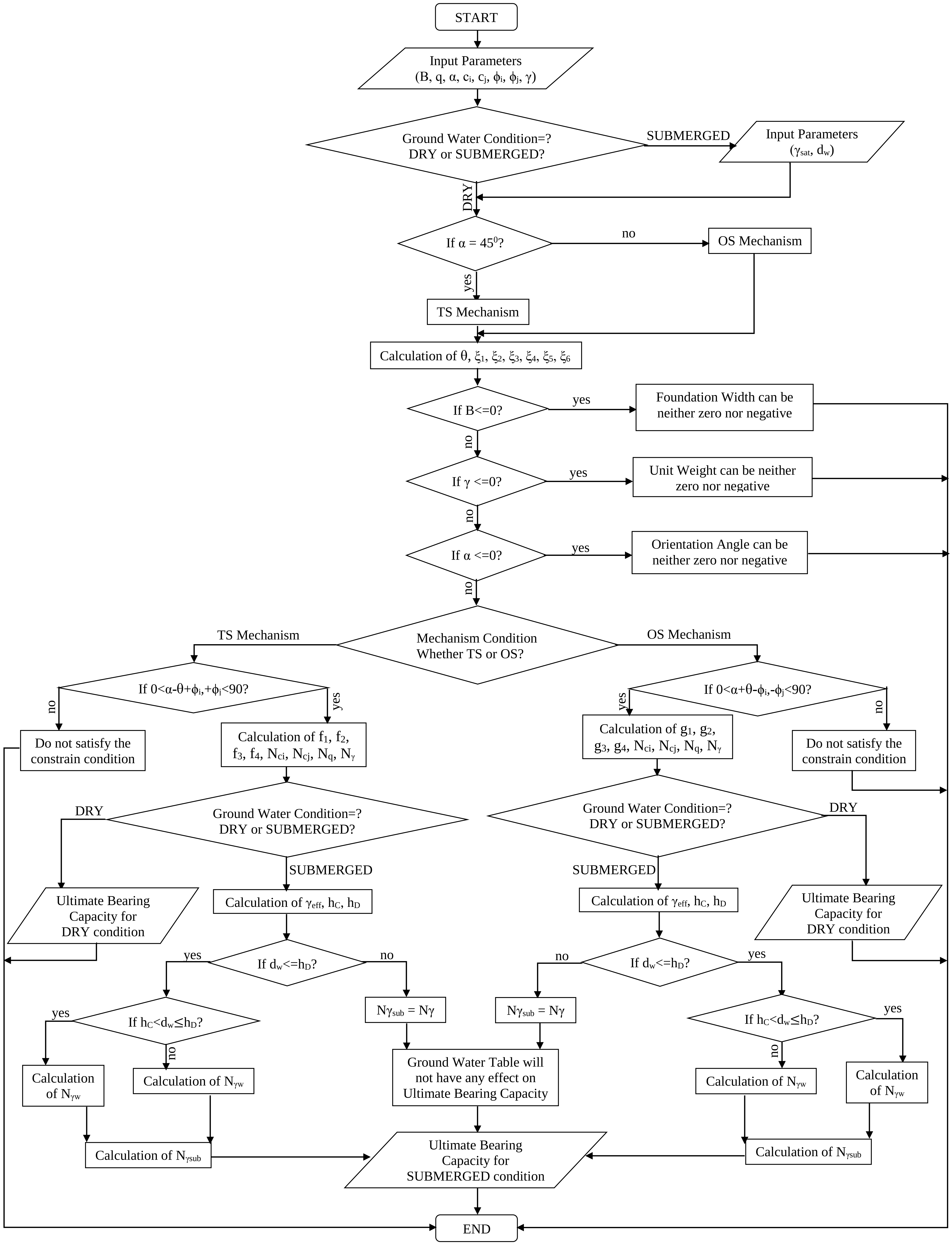


Fig. 4.1: Flowchart of Algorithmic Model for determination of Ultimate Bearing Capacity

The algorithmic model for determination of ultimate bearing capacity of rock mass developed in MATLAB script is given below –

```
%Algorithmic model for determination of Ultimate Bearing Capacity of Rock Foundation.
%Model INPUT parameters are Foundation Width, Surcharge, Orientation Angle, Rock Mass Strength Properties, Physical Properties, Water Table Location.
%Model OUTPUT parameter is Ultimate Bearing Capacity of Rock Foundation.
clc
clear all
format compact
disp ('Welcome to the Algorithmic model for determination of Ultimate Bearing Capacity of Rock Foundation')
disp ('Please enter the magnitudes of the following INPUT Parameters one by one')
% INPUT PARAMETERS
B=input('Width of Foundation (in m)=');
q=input('Surcharge magnitude (in kPa)=');
alpha = input ('Orientation Angle (in degree)=');
ci=input('Cohesion of Intact rock (in kPa)=');
cj=input('Cohesion of Jointed rock (in kPa)=');
phii = input ('Angle of Friction of Intact rock (in degree)=');
phij = input ('Angle of Friction of Jointed rock (in degree)=');
r=input('Bulk Unit Weight (in kN/m^3)=');
disp('Type only the word in Capital letters')
GWC=input('Ground Water condition,(DRY or SUBMERGED)=','s');
switch GWC
   case 'SUBMERGED'
      rsat=input ('Saturated Unit Weight of Submerged Rock (in kN/m^3)=');
      disp('Water Table Location should be Measured from Ground Level');
      dw=input ('Depth of Water (in m)=');
end
if alpha == 45;
   mechanism = 'TS';
else mechanism = 'OS';
```

```
end
clc
% CALCULATION PART
n0 = 100;
s1 = 0.039;
theta = atand(n0*s1/(B*cosd(alpha)))-alpha;
xi1=alpha+theta-phii-phij;
xi2=alpha+theta-phii+phij;
xi3=alpha+phij;
xi4=alpha+theta;
xi5=phii-theta;
xi6=alpha-phij;
if B<=0;
    disp('Foundation Width can be neither zero nor negative. Please try again with correction of Foundation Width.')
elseif r<=0;
    disp('Unit Weight can be neither zero nor negative. Please try again with correction of Unit Weight.')
elseif alpha<=0;
    disp('Orientation Angle can be neither zero nor negative. Please try again with correction of Orientation Angle.')
elseif B>0&&r>0&&alpha>0;
    switch mechanism
        case 'TS'
            if 0>(alpha-theta+phii+phij)||(alpha-theta+phii+phij)>90;
                disp ('Do not satisfy the constrain condition.')
            elseif 0<(alpha-theta+phii+phij)&&(alpha-theta+phii+phij)<90;
                f1=sind(xi6)-sind(xi5)*cosd(xi1);
                f2=-sind(xi6)*sind(xi5)+cosd(xi1);
                f3=sind(2*phij)*cosd(xi2)+sind(xi1);
                f4=sind(2*phij)+cosd(xi2)*sind(xi1);
                Ncj=2*cosd(phij)*cosd(alpha)/f1*((cosd(xi5))^2 + f2/f3*tand(xi4)*((sind(xi2))^2 + f4/tand(alpha)));
```

```
Nci=f2/f1*2*cosd(phii)*cosd(alpha)/cosd(xi4);
Nq=f2*f4/(f1*f3)*2*sind(xi3)*(tand(xi4))/tand(alpha);
Nr=cosd(alpha)*(2*f2/f1*cosd(alpha)*tand(xi4)*(sind(xi5) + f4/f3*sind(xi3)*tand(xi4)/tand(alpha)) - sind(alpha));
switch GWC
    case 'DRY'
        qu=cj*Ncj+ci*Nci+q*Nq+0.5*r*B*Nr;
        qu=qu/1000;
        fprintf ('Ultimate Bearing Capacity of Rock Foundation is %5.3f MPa for \n %3.1f m Foundation Width, %4.1f kPa Surcharge, %4.1f degree Orientation Angle, \n %5.1f kPa Cohesion and %5.2f degree Angle of Friction of Intact Rock, \n %4.1f kPa Cohesion and %5.2f degree Angle of Friction of Jointed Rock and \n %5.2f kN/m^3 Bulk Unit weight. \n', qu,B,q,alpha,ci,phii,cj,phij,r)
    case 'SUBMERGED'
        rw=9.807;
        reff=rsat-rw;
        hC=B*sind(alpha)*cosd(alpha);
        hD=B*(cosd(alpha))^2*tand(alpha+theta);
        if dw<=hD;
            if dw<=hC;
                Nrw=2*dw/(B*sind(2*alpha))*(1+2*f2/f1*sind(xi5) - 2*f2*f4/(f1*f3)*sind(xi3)) +4*f2*f4/(f1*f3)*sind(xi3)*tand(xi4)/tand(alpha) - 2;
            elseif hC<dw&&dw<=hD;
                Nrw=4*f2/f1*tand(xi4)*cosd(alpha)*(cosd(xi4)*sind(xi5)/sind(theta) + f4*sind(xi3)/(f3*sind(alpha))) - 2*dw*f2/(B*f1*cosd(alpha)) *(cosd(xi4)*sind(xi5)/sind(theta) + f4*sind(xi3)/(f3*sind(alpha))) + B*cosd(alpha)/dw*(2*f2/f1*cosd(alpha)*tand(xi4)*sind(xi5)*(1-cosd(alpha)*sind(xi4)/sind(theta)) - sind(alpha));
            end
            Nrsub=reff*Nr/r+dw/B*(1-reff/r)*Nrw;
        elseif dw>hD;
```

```
                        Nrsub=Nr;
                        fprintf ('Note: Ground Water Table will not have any effect on Ultimate Bearing Capacity. \n')
                    end
                    qu=cj*Ncj+ci*Nci+q*Nq+0.5*r*B*Nrsub;
                    qu=qu/1000;
                    fprintf ('Ultimate Bearing Capacity of Rock Foundation is %5.3f MPa for \n %3.1f m Foundation Width, %4.1f kPa Surcharge, %4.1f degree Orientation Angle, \n %5.1f kPa Cohesion and %5.2f degree Angle of Friction of Intact Rock, \n %4.1f kPa Cohesion and %5.2f degree Angle of Friction of Jointed Rock,\n %5.2f kN/m^3 Bulk Unit weight, %5.2f kN/m^3 Saturated Unit weight, %3.1f m Water Table from GL.\n',qu,B,q,alpha,ci,phii,cj,phij,r,rsat,dw)
            end
        end
    case 'OS'
        if 0>(alpha+theta-phii-phij) || (alpha+theta-phii-phij)>90;
            disp ('Do not satisfy the constrain condition.')
        elseif 0<(alpha+theta-phii-phij)&&(alpha+theta-phii-phij)<90;
            g1=cosd(xi1)*sind(xi2)-sind(2*phij);
            g2=cosd(xi1)-sind(xi2)*sind(2*phij);
            g3=sind(2*phij)*cosd(xi2)+sind(xi1);
            g4=sind(2*phij)+cosd(xi2)*sind(xi1);
            Ncj=cosd(phij)*cosd(alpha)/cosd(xi3)*(tand(alpha) + (cosd(xi2))^2/g1 + g2/(g1*g3) *tand(xi4)*((sind(xi2))^2 + g4/tand(alpha)));
            Nci=g2/g1*cosd(phii)*cosd(alpha)/(cosd(xi3)*cosd(xi4));
            Nq=(g2*g4)/(g1*g3)*tand(xi3)*tand(xi4)/tand(alpha);
            Nr=cosd(alpha)*(g2/g1*cosd(alpha) *tand(xi4)/cosd(xi3) *(sind(xi5) + g4/g3 *sind(xi3) *tand(xi4)/tand(alpha)) - sind(alpha));
            switch GWC
                case 'DRY'
                    qu=cj*Ncj+ci*Nci+q*Nq+0.5*r*B*Nr;
                    qu=qu/1000;
```

```
        fprintf ('Ultimate Bearing Capacity of Rock Foundation is %5.3f MPa for \n %3.1f m Foundation Width, %4.1f kPa Surcharge, %4.1f degree Orientation Angle, \n %5.1f kPa Cohesion and %5.2f degree Angle of Friction of Intact Rock, \n %4.1f kPa Cohesion and %5.2f degree Angle of Friction of Jointed Rock and \n %5.2f kN/m^3 Bulk Unit weight. \n', qu,B,q,alpha,ci,phii,cj,phij,r)
    case 'SUBMERGED'
        rw=9.807;
        reff=rsat-rw;
        hC=B*sind(alpha)*cosd(alpha);
        hD=B*(cosd(alpha))^2*tand(alpha+theta);
        if dw<=hD;
            if dw<=hC;
                Nrw=2*dw/(B*sind(2*alpha))*(1+g2*sind(xi5)/(g1* cosd(xi3)) - g2*g4/(g1*g3)*tand(xi3)) + 2*g2*g4/(g1*g3) *tand(xi3) *tand(xi4)/tand(alpha) - 2;
            elseif hC<dw&&dw<=hD;
                Nrw=2*g2/g1*sind(xi4)*cosd(alpha)/cosd(xi3) *(sind(xi5)/sind(theta) + g4*sind(xi3)/(g3*sind(alpha)*cosd(xi4))) - dw*g2/(B*g1*cosd(alpha)*cosd(xi3))*(cosd(xi4) *sind(xi5)/sind(theta) + g4*sind(xi3)/(g3*sind(alpha))) + B*cosd(alpha)/dw *(g2/g1*cosd(alpha)*tand(xi4) *sind(xi5)/cosd(xi3)* (1-cosd(alpha)*sind(xi4)/sind(theta))-sind(alpha));
            end
            Nrsub=reff*Nr/r+dw/B*(1-reff/r)*Nrw;
        elseif dw>hD;
            Nrsub=Nr;
            fprintf ('Note: Ground Water Table will not have any effect on Ultimate Bearing Capacity. \n')
        end
        qu=cj*Ncj+ci*Nci+q*Nq+0.5*r*B*Nrsub;
        qu=qu/1000;
```

```
122.            fprintf ('Ultimate Bearing Capacity of Rock Foundation is %5.3f MPa for \n %3.1f m Foundation Width, %4.1f kPa Surcharge, %4.1f degree Orientation Angle, \n %5.1f kPa Cohesion and %5.2f degree Angle of Friction of Intact Rock, \n %4.1f kPa Cohesion and %5.2f degree Angle of Friction of Jointed Rock,\n %5.2f kN/m^3 Bulk Unit weight, %5.2f kN/m^3 Saturated Unit weight, %3.1f m Water Table from GL.\n', qu,B,q,alpha,ci,phii,cj,phij,r,rsat,dw)
123.          end
124.        end
125.    end
126. end
```

4.1.3 IMPLEMENTATION DETAILS

It is already mentioned that by using the above algorithmic MATLAB program, with known footing width (B), orientation angle (α), rock mass strength properties (c_i, c_j, ϕ_i, ϕ_j), physical properties (γ, γ_{sat}), water table location (d_w), surcharge (q) ultimate bearing capacity of rock mass can be determined in MATLAB software. As the algorithmic model is composed of MATLAB language, it is quite hard to understand it without knowing MATLAB language. Therefore the complete process of the model execution or the statements used in model are discussed below:

Step 1: First three lines are the comment lines that describe about the model, i.e. model purpose, Model input parameters and Model output parameter. These comment lines are not mandatory as they never execute, but it is often necessary to provide information about the algorithm to the user. These comment lines can be viewed in command window with the help of 'help' command. Suppose the model is saved as 'ModelBCR' (BCR stands for Bearing Capacity of Rock), then whenever we execute 'help ModelBCR' command, those three lines will be displayed. Fig. 4.2 shows a view of displaying comment lines of ModelBCR in MATLAB software. Next three lines, i.e. statement lines from 4th to 6th formats the command window for display appearance. For a clear and safe run or execution, display should be blank and there should not be any pre-assigned values to any variable. Statement lines 7th and 8th provide basic information for further execution of model to the user.

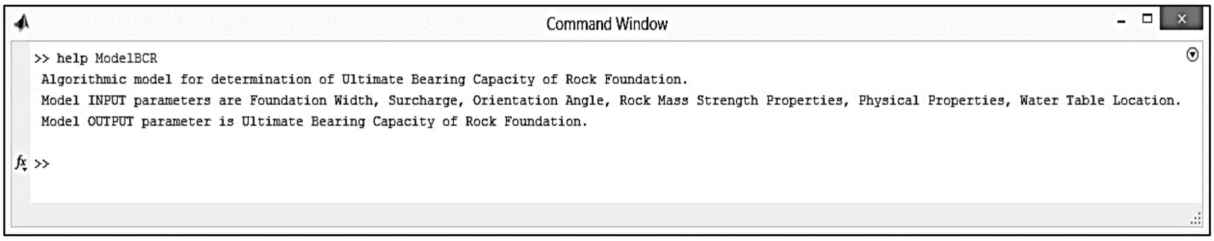


Fig. 4.2: MATLAB Command Window showing comment lines of ModelBCR

Step 2: Statement lines from 10^{th} to 17^{th} are the statements for input parameter. In this step, the input parameters B, q, α, c_i, c_j, ϕ_i, ϕ_j and γ will be asked to insert for further operations.

Step 3: Statement lines from 18^{th} to 25^{th} specify the Ground Water Condition. It will be asked to specify the ground water condition, whether it is 'DRY' or 'SUBMERGED'. This step contains a simple conditional program such as if it is specified the condition as 'DRY', statement lines from 20^{th} to 25^{th} will be skipped. On the other hand, for 'SUBMERGED' condition statement lines from 20^{th} to 25^{th} will be executed and the input parameters γ_{sat}, d_w will be asked to insert.

Step 4: Statement lines from 26^{th} to 29^{th} contains a simple program with conditional statement. Program collects information as mentioned in step 2, more precisely, value of α from user input and specifies the mechanism condition whether 'two sided' or 'one sided' for further execution.

Step 5: Statement lines from 32^{nd} to 40^{th} comprises of theorem equations to calculate θ, ξ_1, ξ_2, ξ_3, ξ_4, ξ_5, ξ_6.

Step 6: Statement line 41^{st} collects information as mentioned in step 2 that B is equal to zero or negative or positive. If B is either negative or zero then a message will be displayed "Foundation Width can be neither zero nor negative. Please try again with correction of Foundation Width." and further operations will be skipped to the end. For positive values of B, execution of 42^{nd} line will be skipped and will be proceed to the next step.

Step 7: Statement line 43^{rd} collects information as mentioned in step 2 that γ is equal to zero or negative or positive. If γ is either negative or zero then a message will be displayed "Unit Weight can be neither zero nor negative. Please try again with correction of Unit Weight." and further operations will be skipped to the end. For positive values of γ, execution of 44^{th} line will be skipped and will be proceed to the next step.

Step 8: Statement line 45th collects information as mentioned in step 2 that α is equal to zero or negative or positive. If α is either negative or zero then the program will display a message will be displayed "Orientation Angle can be neither zero nor negative. Please try again with correction of Orientation Angle." and further operations will be skipped to the end. For positive values of α, execution of 46th line will be skipped and will be proceed to the next step.

Step 9: Statement lines from 48th to 126th will be executed only when the conditions in step 6, step 7 and step 8 are satisfied i.e. for positive values of B, γ and α. These lines contain the main calculation part for ultimate bearing capacity determination. There are several nested program within this step. As the execution proceeds with positive values of B, γ and α, 48th line statement will collect information about mechanism condition as mentioned in step 4 and thereafter it is decided, the commands under decided mechanism will be executed i.e. statement lines either from 49th to 86th or from 87th to 125th. It should be noted that statement lines either from 49th to 86th or from 87th to 125th, algorithm structure is same, but theorem equations or conditions are different. Statement lines from 49th to 86th comprises of theorem equations or conditions for Two-sided mechanism. On the other hand, statement lines from 87th to 125th comprises of theorem equations or conditions for One-sided mechanism. Suppose $\alpha = 45^0$, which will support two-sided mechanism and therefore statement lines from 49th to 86th will execute.

Statement line 50th will verify the constrain condition of two sided theorem with input data. If the constrain condition is satisfied then 51st line will be skipped and further operations of calculating f_1, f_2, f_3, f_4, N_{ci}, N_{cj}, N_q, N_γ will be proceed, otherwise the computation will be ended with a display message "Do not satisfy the constrain condition".

After calculation of f_1, f_2, f_3, f_4, N_{ci}, N_{cj}, N_q, N_γ further computations will be carried out by statement lines from 61st to 85th under nested program for ground water condition. If user mentioned 'DRY' condition in Step 3 then statement lines from 63rd to 65th will be executed and then ultimate bearing capacity will be calculated and further operations will be terminated.

But if user mentioned 'SUBMERGED' condition in Step 3 then statement lines from 63rd to 65th will be skipped and statement lines from 67th to 84th will be executed. It should be noted that in most calculations, unit weight of water is generally taken as either 9.81 kN/m^3 or 10kN/m^3, but for more perfection in result of ultimate bearing capacity, unit weight of water is taken as 9.807kN/m^3. In 'SUBMERGED' condition, firstly, γ_{eff}, h_C, h_D will be calculated by statement lines from 67th to 70th and then proceed to nested program for different location of ground water.

Statement line 71st will decide whether the water table is above the critical point or below the critical point. It is obvious that if location of water table is below the critical point then ground water will have no effect on bearing capacity. If the conditional statement in statement line 71st confirms that water table is below the critical point, i.e. $d_w > h_D$, execution of statement lines from 72nd to 77th will be skipped and operations will be carried out as specified in statement lines 79th and 80th, which will add a display message 'Note: Ground Water Table will not have any effect on Ultimate Bearing Capacity.'. On the other hand, if water table is above or just at critical point i.e. $d_w \leq h_D$ e.g. in case of currently running problem, statement lines from 72nd to 77th will be executed and 79th and 80th line will be skipped. As the model proceeds to line 72nd, nested program of calculation of $N_{\gamma w}$ for different ground water condition will execute. If the conditional statement in 72nd line is satisfied then 73rd line will be executed and execution of 75th line will be skipped. On the other hand, if the conditional statement in 72nd line is not satisfied then execution of 73rd line will be skipped and with compulsion, model will satisfy the condition in 74th and 75th statement lines will be executed. After calculation of $N_{\gamma w}$ for either condition between two mentioned conditions, 77th line statement will be executed and thereafter $N_{\gamma sub}$ will be calculated. After calculation of $N_{\gamma sub}$ for 'SUBMERGED' condition, ultimate bearing capacity will be calculated by executing the command lines 82nd and 83rd. Statement line 84th will display the final output i.e. ultimate bearing capacity of rock foundation with the specified input parameters.

As it is already mentioned in 1st paragraph of step 9, this chapter, same subsection, that algorithm structure is same for statement lines from 49th to 86th or for statement lines from 87th to 125th. Therefore when the orientation angle is other than 45^0 or $\alpha \neq 45^0$, it will follow one-sided mechanism and therefore statement lines from 87th to 125th will execute. These statement lines will follow the same procedure as followed by statement lines from 49th to 86th, but each theorem calculations and conditions are a bit different, such as constrain condition is different, calculations of g_1, g_2, g_3, g_4 will be carried out instead of f_1, f_2, f_3, f_4.

4.1.4 EXECUTION STEPS

Before execution, first of all, the complete algorithm must be saved in a MATLAB script file. This task can be easily done by choosing 'Save As' option from 'EDITOR' tab in MATLAB or simply and commonly by pressing 'Ctrl+S' key, inbuilt shortcut key. It should be ensured that the file name with which the file is saved must satisfy the MATLAB conditions (must begin with a letter, can include digits and underscore but no spaces, and up to 63 characters long and others) for writing a file name. It is very essential that user must check whether the file is located in 'Current Folder' or not. If not, then either the MATLAB file directory have to change to the location where the file is saved or the file have to be moved to directory location. After completing the above steps user may execute the complete model by choosing 'Run' option from 'Editor' tab or by typing the file name in command window or by simply pressing 'F5' key, inbuilt shortcut key. As the model run user must proceed the following steps:

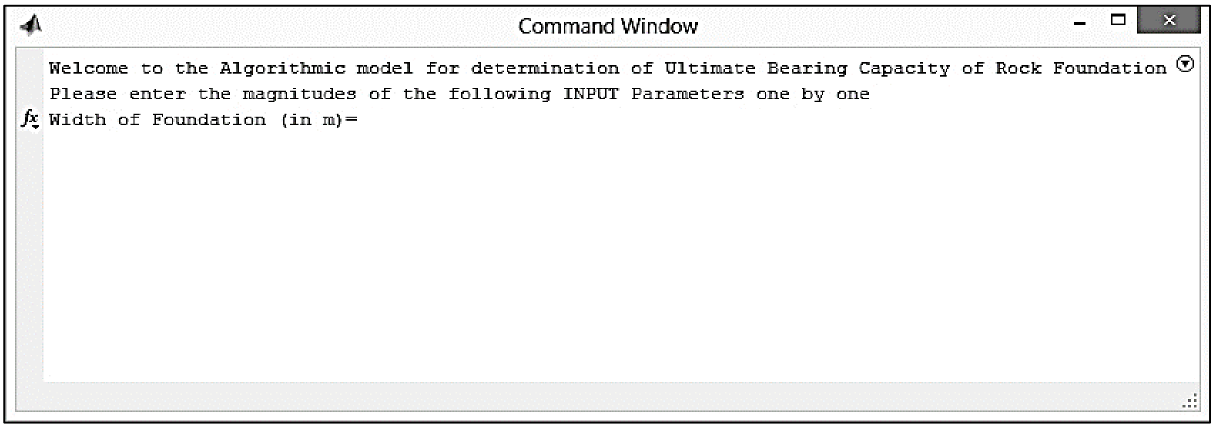


Fig. 4.3: MATLAB Command Window, asking input of foundation width.

Step 1: In command window three lines will appear. First line describes the model purpose, second line notifies to put the input parameters and third line asks to insert input of 'Width of Foundation (in m)'. Fig. 4.3 shows a view of 'MATLAB Command Window' of first step execution. Suppose width of foundation is 1m, as stated in 2nd line, only magnitude have to be entered, therefore input will be 1.

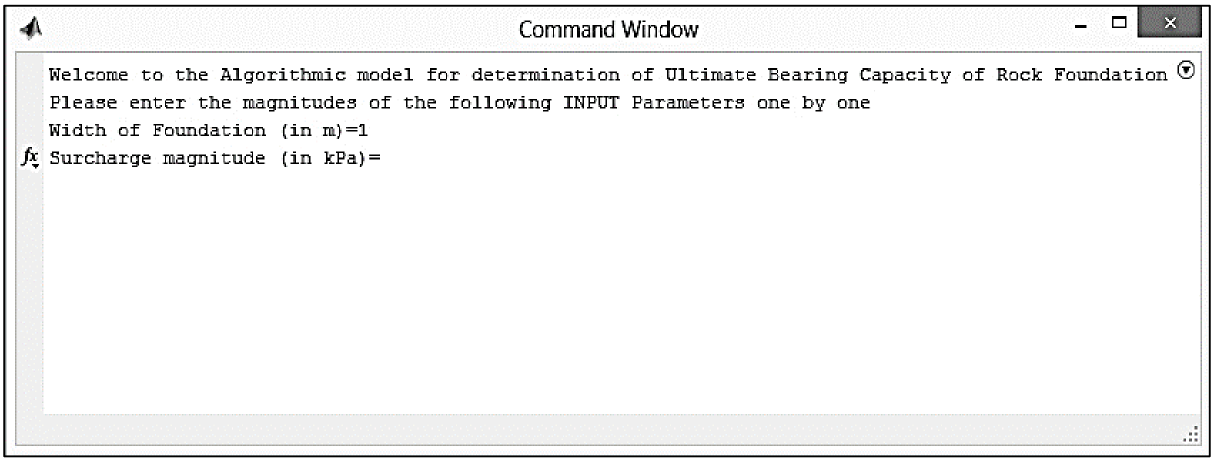


Fig. 4.4: MATLAB Command Window, asking input of surcharge magnitude.

Step 2: After entering the input value of foundation width, next line appears asking to insert input of 'Surcharge magnitude' (in kPa). Fig. 4.4 shows a view of 'MATLAB Command Window' of second step execution. Suppose surcharge magnitude is 20kPa, then input will be 20.

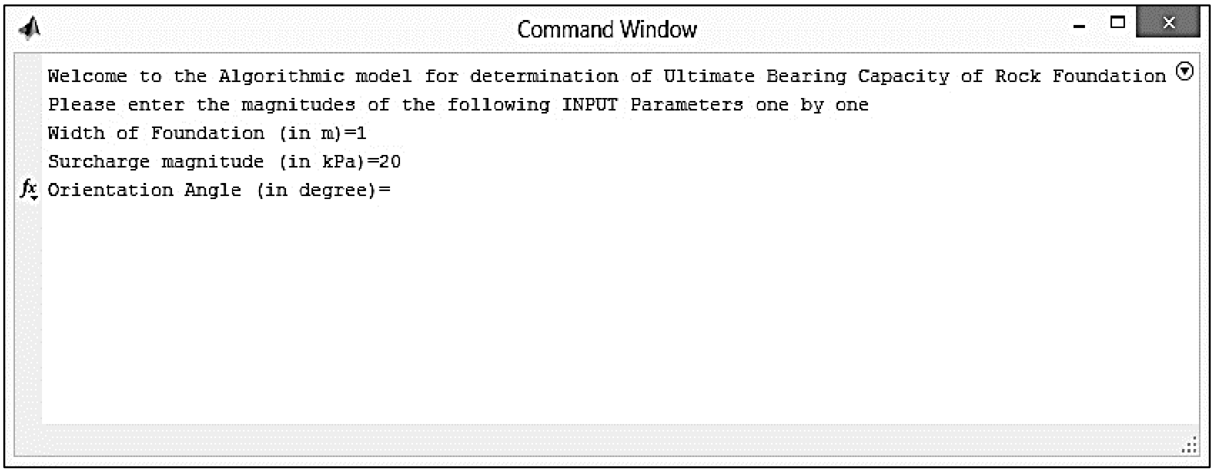


Fig. 4.5: MATLAB Command Window, asking input of orientation angle.

Step 3: After entering the input value of surcharge magnitude, next line appears asking to insert input of 'Orientation Angle' (in degree). Fig. 4.4 shows a view of 'MATLAB Command Window' of third step execution. Suppose orientation angle is 45^0, then input will be 45.

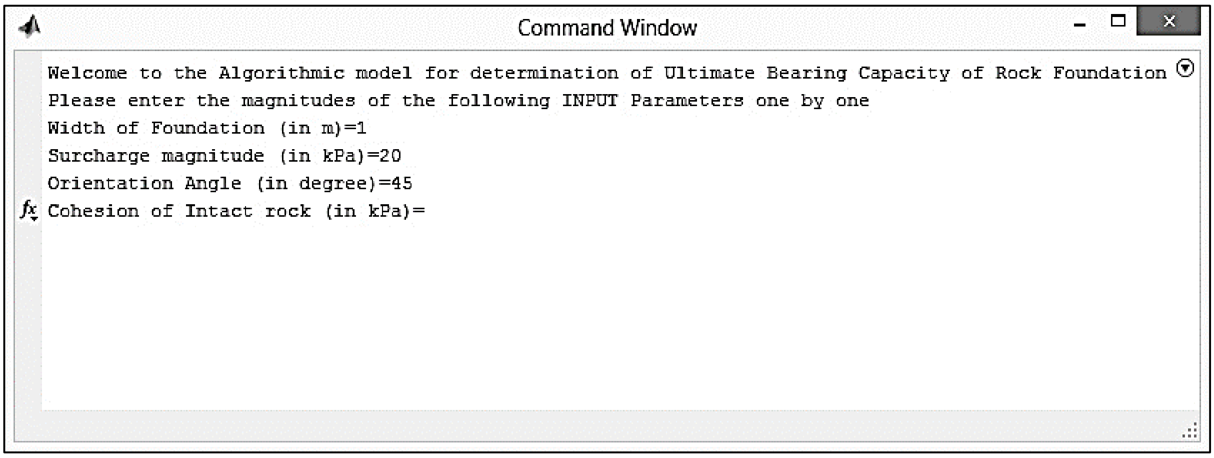


Fig. 4.6: MATLAB Command Window, asking input of cohesion of intact rock.

Step 4: After entering the input value of orientation angle, next line appears asking to insert input of ‘Cohesion of Intact rock’ (in kPa). Fig. 4.6 shows a view of ‘MATLAB Command Window’ of fourth step execution. Suppose cohesion of intact rock is 5MPa, as the input is asked to enter in kPa unit therefore input will be 5000.

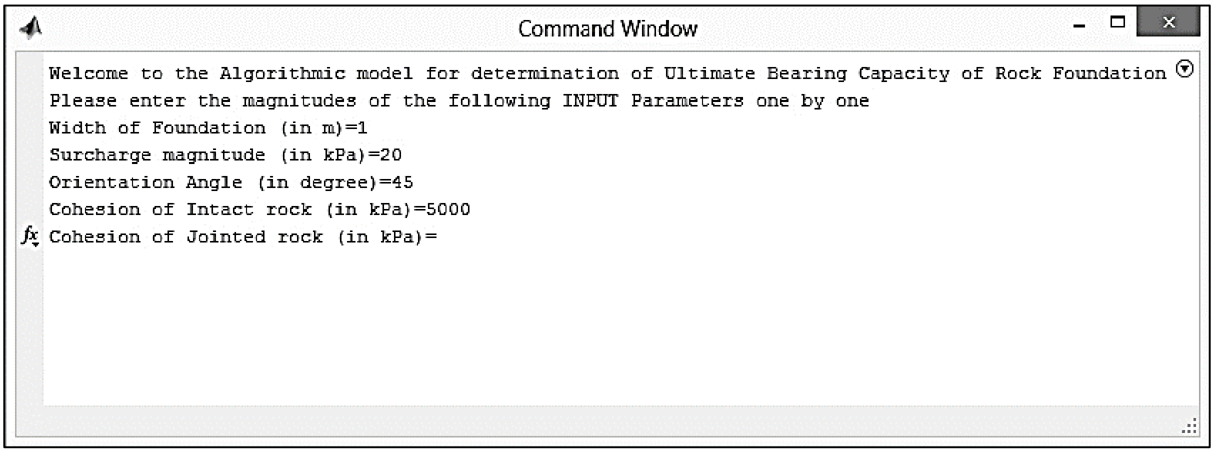


Fig. 4.7: MATLAB Command Window, asking input of cohesion of jointed rock.

Step 5: After entering the input value of cohesion of intact rock, next line appears asking to insert input of ‘Cohesion of Jointed Rock’ (in kPa). Fig. 4.7 shows a view of ‘MATLAB Command Window’ of fifth step execution. Suppose cohesion of jointed rock is 50kPa, then input will be 50.

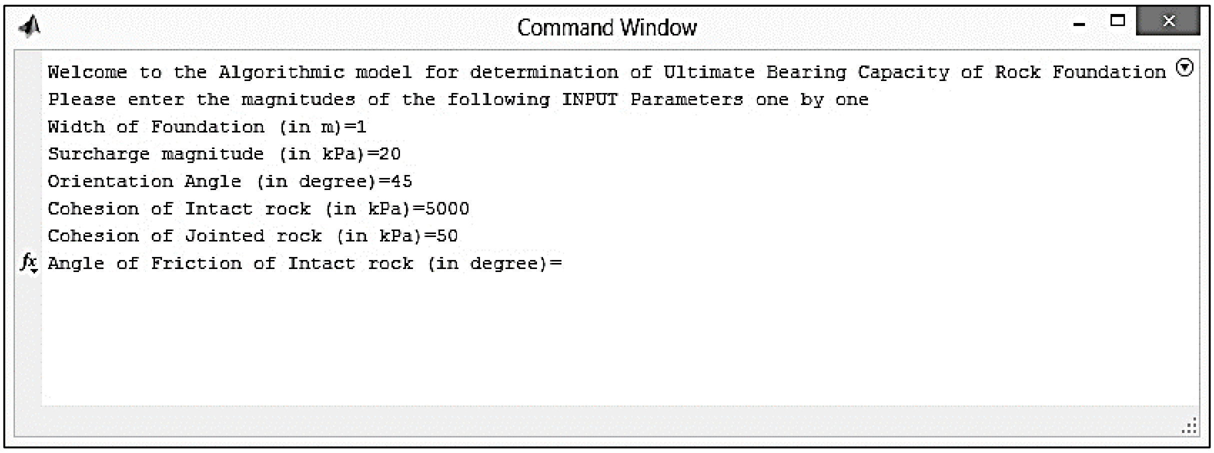


Fig. 4.8: MATLAB Command Window, asking input of frictional angle of intact rock

Step 6: After entering the input value of cohesion of jointed rock, next line appears asking to insert input of 'Angle of Friction of Intact Rock' (in degree). Fig. 4.8 shows a view of 'MATLAB Command Window' of sixth step execution. Suppose frictional angle of intact rock is 35^0, then input will be 35.

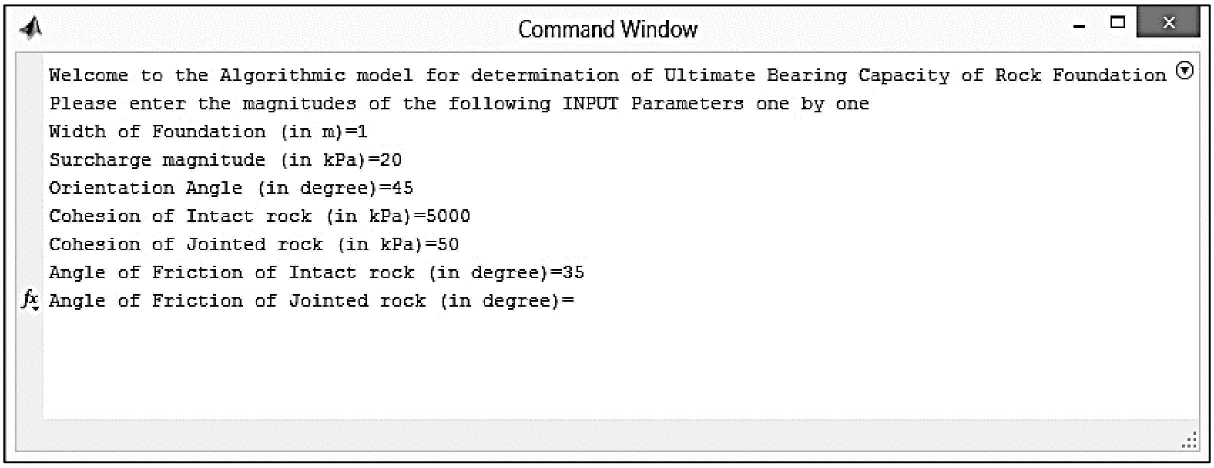


Fig. 4.9: MATLAB Command Window, asking input of frictional angle of jointed rock

Step 7: After entering the input value of angle of friction of intact rock, next line appears asking to insert input of 'Angle of Friction of Jointed Rock' (in degree). Fig. 4.9 shows a view of 'MATLAB Command Window' of seventh step execution. Suppose frictional angle of jointed rock is 35^0, then input will be 35.

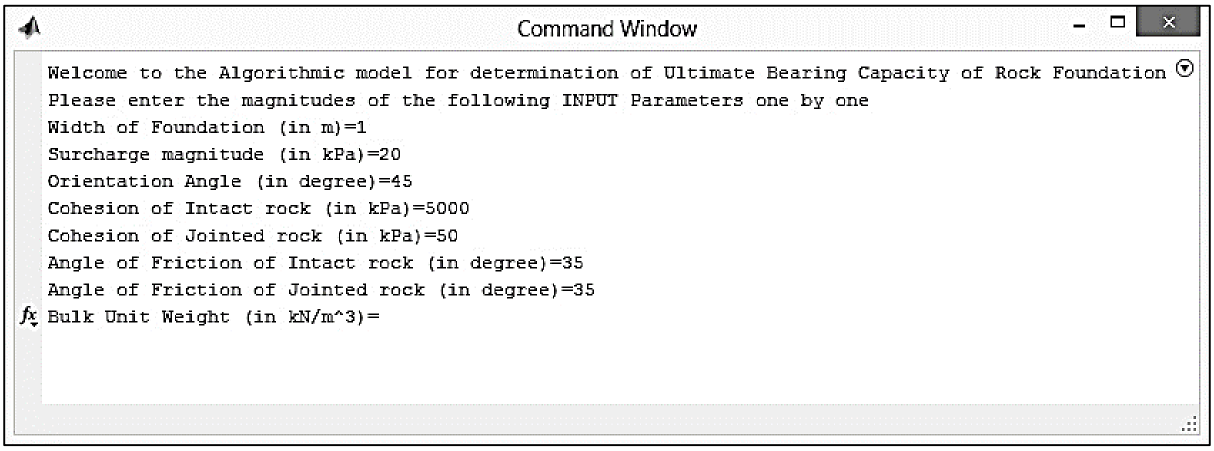


Fig. 4.10: MATLAB Command Window, asking input of Bulk Unit Weight

Step 8: After entering the input value of angle of friction of jointed rock, next line appears asking to insert input of 'Bulk Unit Weight' (in kN/m^3). Fig. 4.10 shows a view of 'MATLAB Command Window' of eighth step execution. Suppose bulk unit weight of rock mass is 27kN/m^3, then input will be 27.

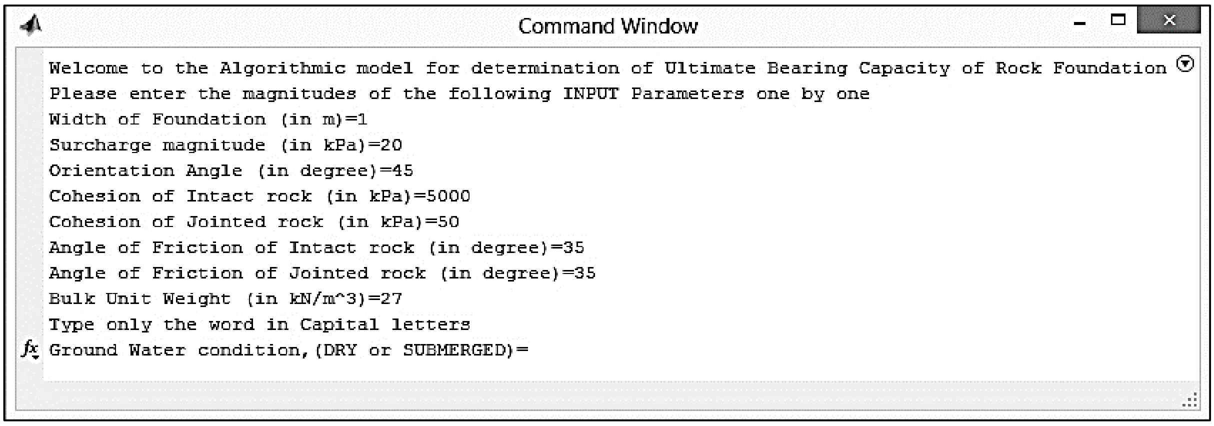


Fig. 4.11: MATLAB Command Window, asking input of Ground water condition

Step 9a: After entering the input value of bulk unit weight, next two lines appear asking to insert input of 'Ground Water Condition' (DRY or SUBMERGED) with a guide message which indicates that input have to type in capital letters. Fig. 4.11 shows a view of 'MATLAB Command Window' of ninth step execution. Suppose there is no water table i.e. ground water condition is dry, then input will be DRY. Fig. 4.12 shows a view of 'MATLAB Command Window' at the verge of completion of ninth step.

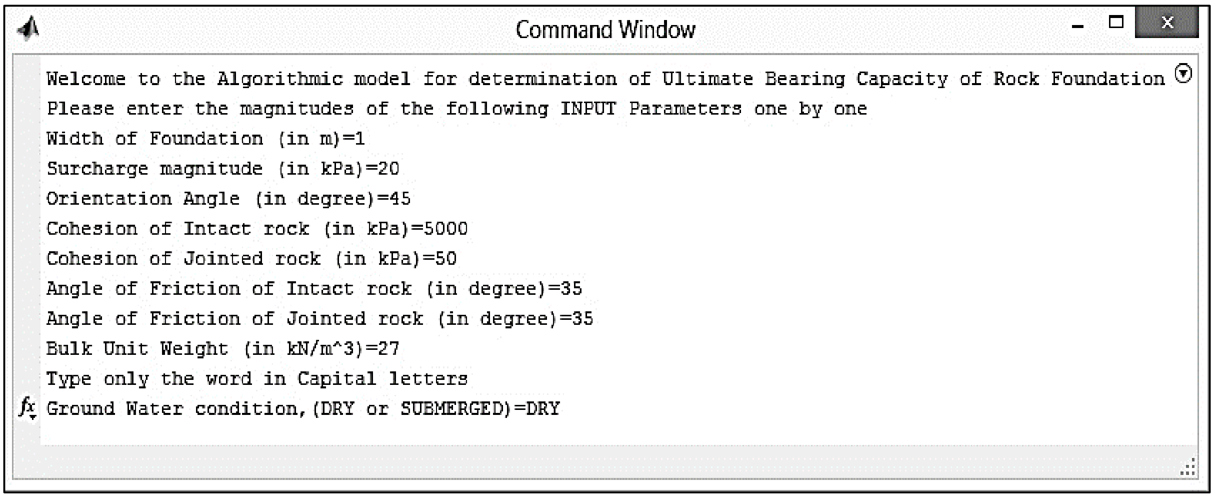


Fig. 4.12: MATLAB Command Window, inserting input as dry condition in Ground water condition

Step 10a: After entering the input value of ground water condition, output will be displayed. First line will show the ultimate bearing capacity of rock foundation, and the next lines will show the input parameters for which bearing capacity is determined. Fig. 4.13 shows a view of 'MATLAB Command Window' displaying output for dry condition.

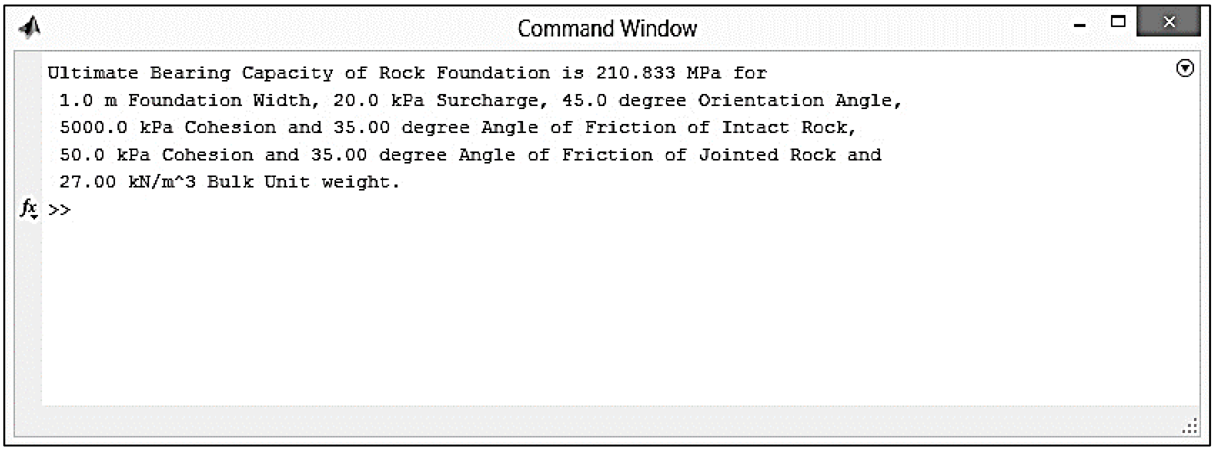
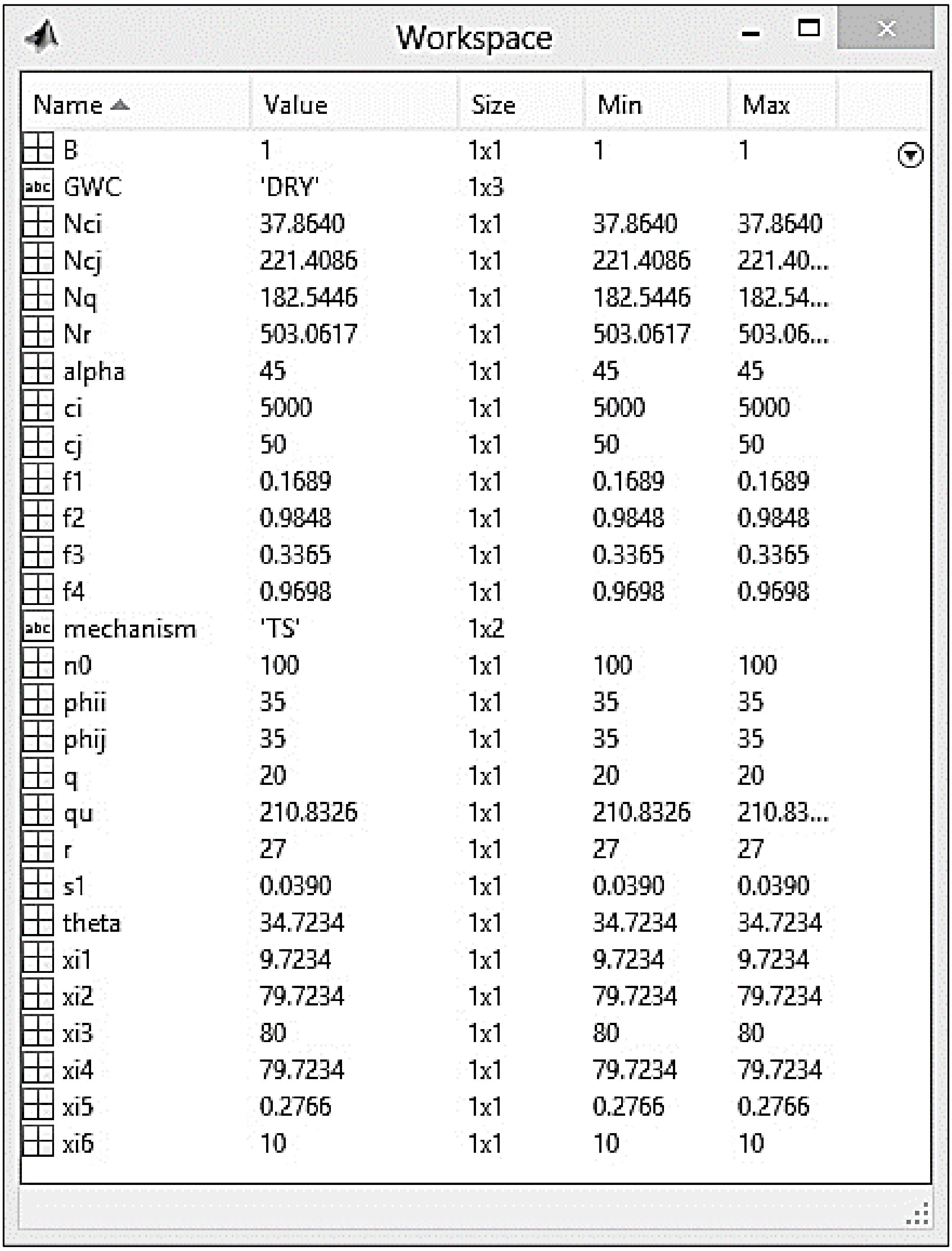


Fig. 4.13: MATLAB Command Window, showing output with its inputs for a simple problem of rock foundation without submergence.

Therefore, for the case of 1 m foundation width, 20 kPa surcharge, 45^0 orientation angle, 5 MPa cohesion of intact rock, 50 kPa cohesion of jointed rock, 35^0 angle of friction for both intact and jointed rock, 27 kN/m^3 bulk unit weight, the model for determination of ultimate bearing capacity yields ultimate bearing capacity 210.833 MPa. The same result obtained by Imani et al. (2012).

Variables addressed in the process of execution were created and stored in workspace. MATLAB workspace provides complete information about the variables i.e. variable name, value, size, minimum and maximum values, statistical descriptors e.g. mean, median, mode, standard deviation and many others. Fig. 4.14 shows a view of MATLAB workspace after completion of execution of Algorithmic Model for dry condition.



Name	Value	Size	Min	Max
B	1	1x1	1	1
GWC	'DRY'	1x3		
Nci	37.8640	1x1	37.8640	37.8640
Ncj	221.4086	1x1	221.4086	221.40...
Nq	182.5446	1x1	182.5446	182.54...
Nr	503.0617	1x1	503.0617	503.06...
alpha	45	1x1	45	45
ci	5000	1x1	5000	5000
cj	50	1x1	50	50
f1	0.1689	1x1	0.1689	0.1689
f2	0.9848	1x1	0.9848	0.9848
f3	0.3365	1x1	0.3365	0.3365
f4	0.9698	1x1	0.9698	0.9698
mechanism	'TS'	1x2		
n0	100	1x1	100	100
phii	35	1x1	35	35
phij	35	1x1	35	35
q	20	1x1	20	20
qu	210.8326	1x1	210.8326	210.83...
r	27	1x1	27	27
s1	0.0390	1x1	0.0390	0.0390
theta	34.7234	1x1	34.7234	34.7234
xi1	9.7234	1x1	9.7234	9.7234
xi2	79.7234	1x1	79.7234	79.7234
xi3	80	1x1	80	80
xi4	79.7234	1x1	79.7234	79.7234
xi5	0.2766	1x1	0.2766	0.2766
xi6	10	1x1	10	10

Fig. 4.14: MATLAB workspace showing variables for the stated problem (Dry Condition)

In many practical cases, presence of ground water may found. For submerged condition, the execution process is slightly different. First eight steps are same as dry condition, but from ninth step, execution process will be different. Therefore, ninth and tenth step are discussed in two different manners which are distinguished by 9a and 9b or 10a and 10b. Steps 9a and 10a are for dry condition while steps 9b and 10b are for submerged condition. Suppose input parameters up to eight step are same, but from ninth step inputs are different due to submerged condition. Therefore, execution from ninth step for submerged condition is discussed below:

Step 9b: After entering the input value of bulk unit weight, next two lines will be appeared asking to insert input of ‘Ground Water Condition’ (DRY or SUBMERGED) with a guide message which indicates that input have to type in capital letters. Fig. 4.15 shows a view of ‘MATLAB Command Window’ after ninth step execution. Suppose for currently running problem, there is availability of water table i.e. ground water condition is submerged, in that case, input will be SUBMERGED.

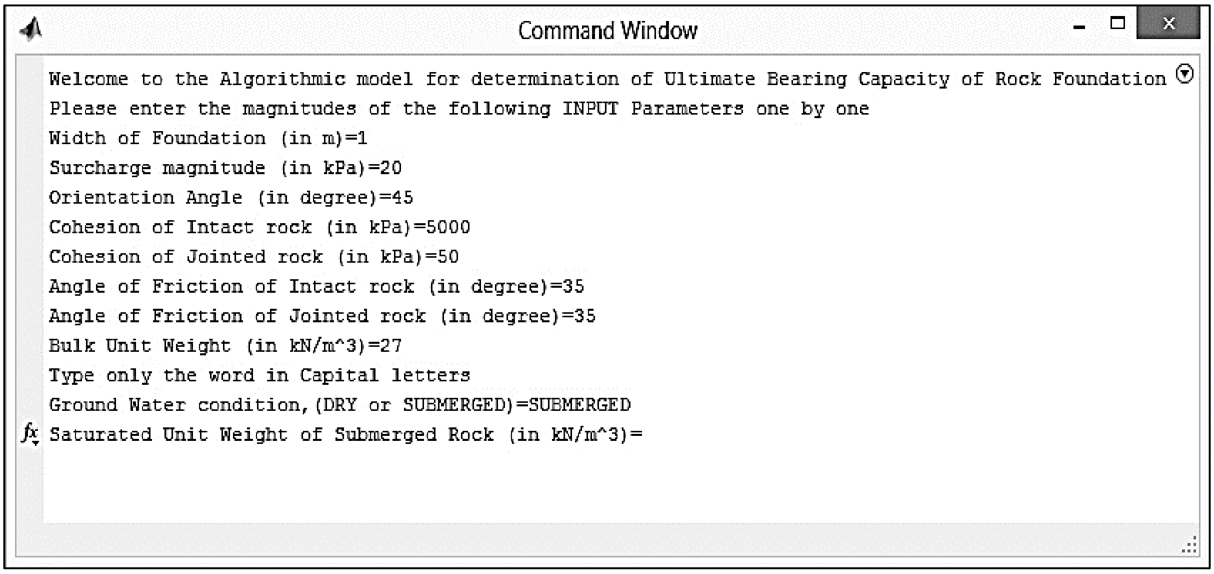


Fig. 4.15: MATLAB Command Window, asking input of saturated unit weight.

Step 10b: After entering the input of ground water condition as submerged, next line appears asking to insert input of ‘Saturated Unit Weight of Submerged Rock’ (in kN/m^3). Fig. 4.15 shows a view of ‘MATLAB Command Window’ of tenth (b) step execution. Suppose saturated unit weight is 30kN/m^3, then input will be 30.

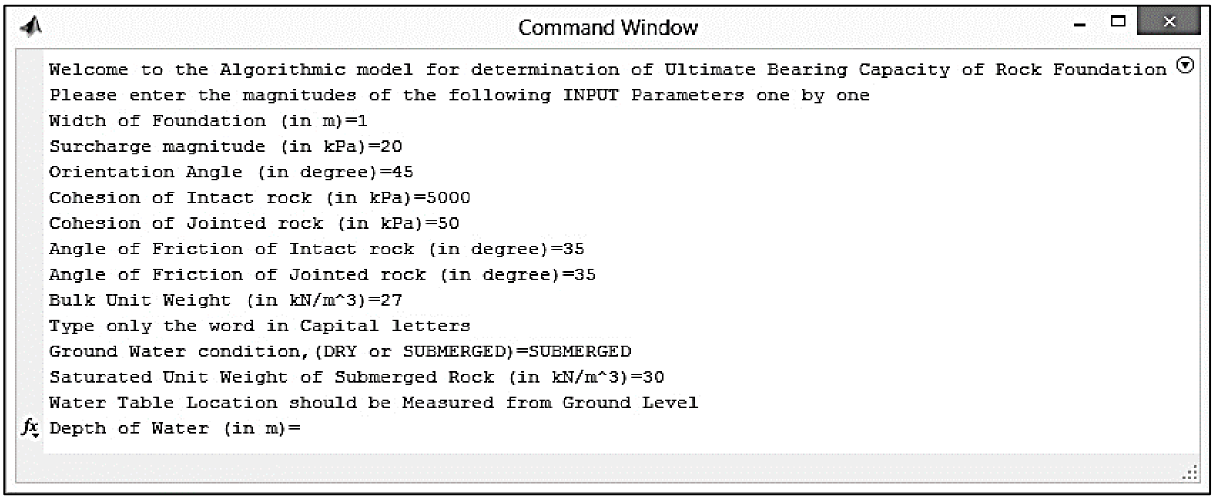


Fig. 4.16: MATLAB Command Window, asking input of depth of water.

Step 11: After entering the input of saturated unit weight, next two lines will be appeared asking to insert input of 'Depth of water' (in m) with a guide message which indicates that water table location should have to be measured from ground level. Fig. 4.16 shows a view of 'MATLAB Command Window' of eleventh step execution. Suppose water table is at depth of 0.4m from ground level, then input will be 0.4.

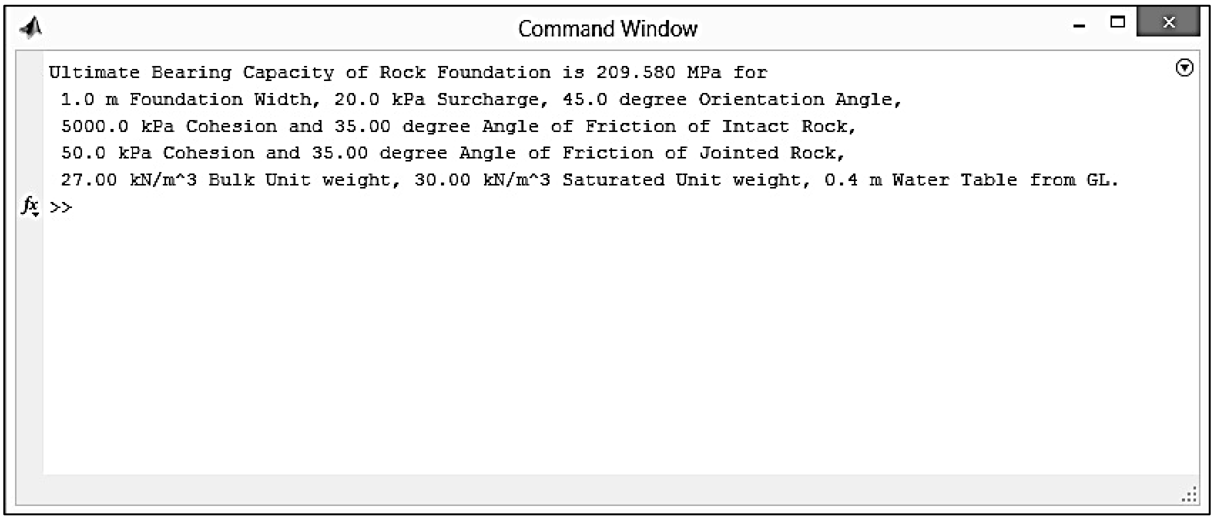


Fig. 4.17: MATLAB Command Window, showing output with its inputs for a simple problem of rock foundation with submergence.

Step 12: After entering the input value of water depth, output will be displayed. First line will show the ultimate bearing capacity of rock foundation, and the next lines will show the input parameters for which bearing capacity is determined. Fig. 4.17 shows a view of 'MATLAB Command Window' displaying output for submerged condition.

Therefore, for the case of 1 m foundation width, 20 kPa surcharge, 45^0 orientation angle, 5 MPa cohesion of intact rock, 50 kPa cohesion of jointed rock, 35^0 angle of friction for both intact and jointed rock, 27 kN/m^3 bulk unit weight, 30 kN/m^3 saturated unit weight and ground water at 40 cm from ground level, the model for determination of ultimate bearing capacity evaluates ultimate bearing capacity as 209.580 MPa.

As like dry condition, for submerged condition also variables addressed in the process of execution were created and stored in workspace. But this time, ground water condition is replaced by submerged condition, and variables associated with submerged condition is newly created and stored in MATLAB workspace. Fig. 4.18 shows a view of undocked MATLAB workspace window after completion of execution of Algorithmic Model for submerged condition.

It has been observed that bearing capacity for dry condition is higher than the bearing capacity for submerged condition. Therefore it can concluded that due to the effect of submergence, bearing capacity is slightly reduced.

Workspace

Name	Value	Size	Min	Max
B	1	1x1	1	1
GWC	'SUBMERGED'	1x9		
Nci	37.8640	1x1	37.8640	37.8640
Ncj	221.4086	1x1	221.4086	221.40...
Nq	182.5446	1x1	182.5446	182.54...
Nr	503.0617	1x1	503.0617	503.06...
Nrsub	410.2649	1x1	410.2649	410.26...
Nrw	337.4567	1x1	337.4567	337.45...
alpha	45	1x1	45	45
ci	5000	1x1	5000	5000
cj	50	1x1	50	50
dw	0.4000	1x1	0.4000	0.4000
f1	0.1689	1x1	0.1689	0.1689
f2	0.9848	1x1	0.9848	0.9848
f3	0.3365	1x1	0.3365	0.3365
f4	0.9698	1x1	0.9698	0.9698
hC	0.5000	1x1	0.5000	0.5000
hD	2.7577	1x1	2.7577	2.7577
mechanism	'TS'	1x2		
n0	100	1x1	100	100
phii	35	1x1	35	35
phij	35	1x1	35	35
q	20	1x1	20	20
qu	209.5798	1x1	209.5798	209.57...
r	27	1x1	27	27
reff	20.1930	1x1	20.1930	20.1930
rsat	30	1x1	30	30
rw	9.8070	1x1	9.8070	9.8070
s1	0.0390	1x1	0.0390	0.0390
theta	34.7234	1x1	34.7234	34.7234
xi1	9.7234	1x1	9.7234	9.7234
xi2	79.7234	1x1	79.7234	79.7234
xi3	80	1x1	80	80
xi4	79.7234	1x1	79.7234	79.7234
xi5	0.2766	1x1	0.2766	0.2766
xi6	10	1x1	10	10

Fig. 4.18: MATLAB workspace showing variables for the stated problem (Submerged Condition)

4.2 EFFECT OF SUBMERGENCE

It is obvious that in presence of water, ultimate bearing capacity will be reduced. But it is quite difficult to say the behavior of reduction i.e. reduction is rapidly growing or gradually growing or it is of uneven nature. Therefore, a concise study on the effect of submergence is carried out to observe the behavior of reduction of ultimate bearing capacity in presence of water. It should be noted that, in this study, water depth is measured from ground level.

4.2.1 ANALYSIS PROCESS

With the help of the algorithmic model presented before, ultimate bearing capacity for different ground water depth can be evaluated. The same set of data analyzed before are used with variation range of ground water depth from 0.0 m to 4.0 m for analysis of effect of submergence in ultimate bearing capacity. Table 4.1 shows the data set for analysis of effect of submergence.

Table 4.1: Data set for analysis of effect of submergence

PARAMETER	VALUE	
	Minimum	Maximum
Width of Foundation, B (m)	1	
Surcharge Magnitude, q (kPa)	20	
Cohesion of Intact Rock, c_i (kPa)	5000	
Cohesion of Jointed Rock, c_j (kPa)	50	
Angle of Friction of Intact Rock, φ_i (degree)	35	
Angle of Friction of Jointed Rock, φ_j (degree)	35	
Orientation Angle, α (degree)	45	
Bulk Unit Weight, γ (kN/m^3)	27	
Saturated Unit Weight, γ_{sat} (kN/m^3)	30	
Water Table from ground, d_w (m)	0.0	4.0

For better analysis, large number of values have been generated and for that, variation scale has been taken with very little difference, therefore, 0.01 m or 1 cm variation scale of ground water depth has been taken into account. Large number of result set has been evaluated, i.e. 401 sets of result comprising of ultimate bearing capacity for each different ground water depth. Graphical representation may be carried out in MATLAB software. But for better visualization, graphical representation is carried out in Microsoft excel spreadsheet. Data generated in MATLAB workspace is sorted in a tabulation form and then exported to excel spreadsheet.

4.2.2 GRAPHICAL REPRESENTATION

Two sets of column comprising of ground water depth and ultimate bearing capacity are represented graphically in the form of scatter graph with ground water depth as horizontal axis and ultimate bearing capacity as vertical axis. Additionally, critical point for each bearing capacity is located as critical line which intersect the ultimate bearing capacity curve. Value of intersecting point is shown nearby vertical axis, pointed by a line created and extended from the point of intersection. A trend line is then created to represent the nearest shape of the curve showing its polynomial equation along with R-squared value. Plot area is filled in gradient form to observe the presence of water table and gradual reduction of the same. Fig. 4.19 represents the behavior of ultimate bearing capacity with gradual reduction of water table.

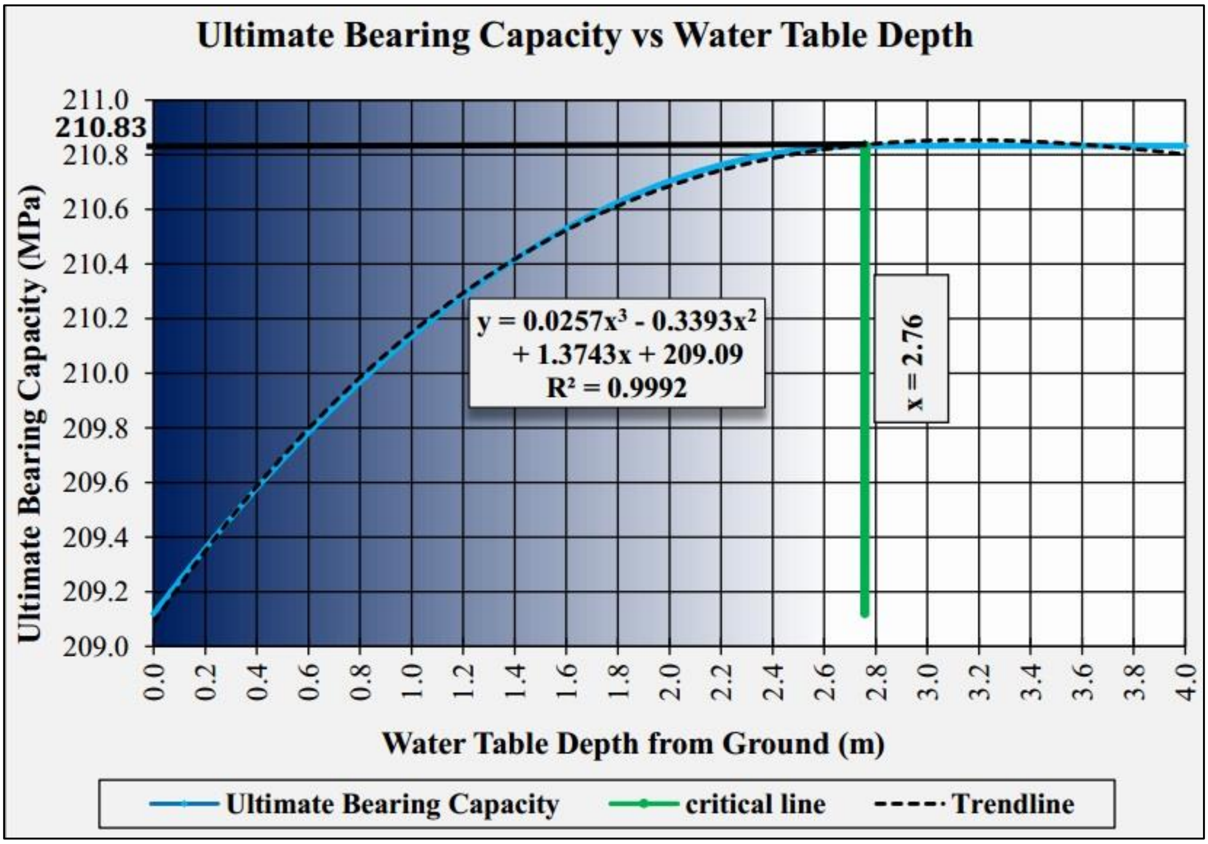


Fig. 4.19: Behavior of Ultimate Bearing Capacity with Submergence.

4.2.3 OBSERVATION

From the graphical representation, it is clearly observed that ultimate bearing capacity changes for the same set of parameters i.e. for same foundation width, surcharge magnitude, orientation angle, strength and physical properties of rock mass, where water table depth is unstable. From fig. 19 it is observed that the ultimate bearing capacity is gradually increasing with the gradual reduction of water table depth. Technically, increment curvature is not purely linear, firstly at initial state, i.e. at high level of water it shows linear behavior with 45^0 gradient, after which it reaches transition state i.e. water table is about to vanish, it shows nonlinear behavior in form of exponentiation and finally at perfectly dry state, it shows a completely linear straight line with no gradient from which it can be concluded that further reduction of water table depth will not have any effect on ultimate bearing capacity.

Therefore, it can be concluded that for reduction of water table depth, keeping other factors constant, ultimate bearing capacity of rock mass will be constantly increased up to the critical point and after that water table will not have any effect on ultimate bearing capacity.

4.3 COMPARISON BETWEEN MECHANISMS

For determination of ultimate bearing capacity, Imani et al. (2012) suggested two different failure mechanism depending upon joint sets, which were already discussed in chapter 3, subsection 'Failure Mechanism'. Condition for choosing each mechanism mainly depends upon the orientation angle of joint sets. Assumption made for failure mechanism clearly indicates that if orientation angle is 45^0, failure mechanism will be 'Two-Sided Failure Mechanism' otherwise it will be considered as 'One-Sided Failure Mechanism'. Suggested theorem by Imani et al. (2012) for determination of ultimate bearing capacity for submerged rock foundation comprises of 32 equations i.e. from eq. 1 to eq. 32 in chapter 3, subsection 'Calculations', out of which 12 equations are common to both failure mechanisms i.e. two sided failure mechanism and one sided failure mechanism, remaining 20 equations are divided into two categories i.e. 10 equations are for each mechanism. Considering all those equations all at a time, makes the computational approach very time consuming and tedious. Hence, an attempt has been made to study regarding the comparison between one sided failure mechanism and two sided failure mechanism. It should be noted that the comparison is done by keeping orientation angle constant at 45^0, whereas the other parameters involved in determination of bearing capacity varies individually.

4.3.1 STOCHASTIC MODEL CREATION

For comparison of failure mechanisms, ultimate bearing capacity is calculated for both one sided and two sided mechanism for orientation angle, 45^0. For determination of ultimate bearing capacity, 10 parameters are involved, wherein, only orientation angle is constant i.e. 45^0. For an ease of calculation, foundation width (B) and surcharge magnitude (q) have been assumed constant, such as 1 m foundation width and 20 kPa surcharge. For deterministic analysis, i.e. out of 7 variable parameters, considering 6 as constant and remaining one varies and repeating the same, computational approach and observation of comparison will create confusion. Therefore, rather than considering a deterministic analysis, a stochastic approach is carried out, where all these parameters vary simultaneously with random values within a specified range. Hence, for randomly varying parameter, coefficient of variation is taken as 5% for strength parameters, 2% for physical properties, and 100% for water table depth into the nominal values used by Imani et al. (2012). Data set used for this approach is enlisted in table 4.2.

Table 4.2: Data set used for comparison between one side & two sided failure mechanisms

PARAMETER	VALUE	
	Minimum	Maximum
Orientation Angle, α (degree)	45	
Width of Foundation, B (m)	1	
Surcharge Magnitude, q (kPa)	20	
Cohesion of Intact Rock, c_i (kPa)	4750	5250
Cohesion of Jointed Rock, c_j (kPa)	47.50	52.50
Angle of Friction of Intact Rock, φ_i (degree)	33.25	36.75
Angle of Friction of Jointed Rock, φ_j (degree)	33.25	36.75
Bulk Unit Weight, γ (kN/m^3)	26.46	27.54
Saturated Unit Weight, γ (kN/m^3)	29.40	30.60
Water Table from ground, d_w (m)	0	4.00

For stochastic approach, a stochastic model has been developed in Microsoft excel spreadsheet. Stochastic model is so called due to the fact of random value generation. Thereafter, theorem equations are formulated in spreadsheet with the help of pre-assigned formulae of excel. Then 5000 rows have been developed such that each row formulated with the same set of equations. It should be noted that every single trail or run will give different set of outputs but graphical representation will be same. Presented study is based on only one trail, but it has been observed that in this case, each trail will evaluate the same. Calculated ultimate bearing capacity varies from 146.83 MPa, lower bound bearing capacity to 346.08 MPa, upper bound bearing capacity. But for present case, stochastic model evaluates bearing capacity ranges from 150.41 MPa to 338.10 MPa, within the range of lower to upper bound bearing capacity. Excel formulae used for this approach are enlisted in Appendix-IV.

4.3.2 GRAPHICAL REPRESENTATION

Two sets of array (column) comprising of ultimate bearing capacity for one sided mechanism and ultimate bearing capacity for two sided mechanism are represented graphically in the form of scatted graph with ultimate bearing capacity for two sided failure mechanism as horizontal axis and ultimate bearing capacity for one sided failure mechanism as vertical axis. A trend line is then created to represent the closest shape of plotted graph showing its equation along with R-squared value. Fig. 4.20 represents the comparison graph of ultimate bearing capacity between one sided and two sided failure mechanism.

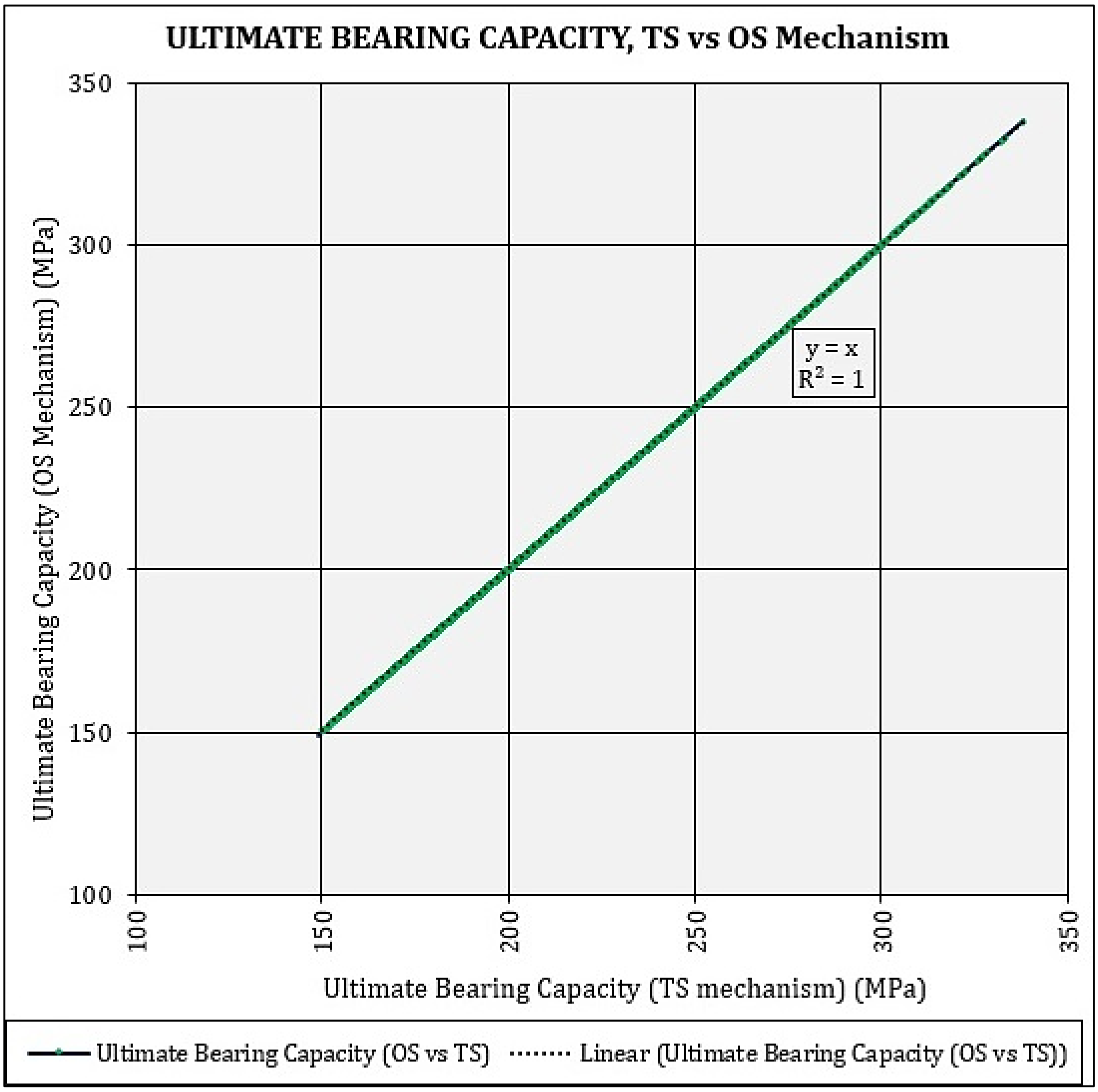


Fig. 4.20: Comparison graph of failure mechanism (Two sided & One Sided)

4.3.3 OBSERVATION

From the graphical representation, it is clearly observed that the ultimate bearing capacity for one sided failure mechanism and two sided failure mechanism are exactly same for uneven variation of theorem parameters keeping orientation angle constant at 45^0. Technically, presented graph shows a purely linear behavior with 45^0 gradient which implies that plotting points have the same axis coordinate. Therefore, theorem equations for two sided failure mechanism can be eliminated for computational approach. In other words, out of 32 theorem equations, 10 equations can be eliminated i.e. eq. 10, 11, 12, 13, 18, 19, 20, 21, 29 and 30 (in chapter 3, subsection 'Calculations') and ultimate bearing capacity can be determined from rest 22 theorem equations. Additionally, the theorem assumption 6 mentioned in chapter 3, subsection 'Assumptions' can be eliminated, as for all condition of orientation angle, ultimate bearing capacity can be determined considering one sided failure mechanism.

Therefore, it can be concluded that the ultimate bearing capacity for submerged rock foundation can be determined by considering only one sided failure mechanism independently.

4.4 RELIABILITY ANALYSIS

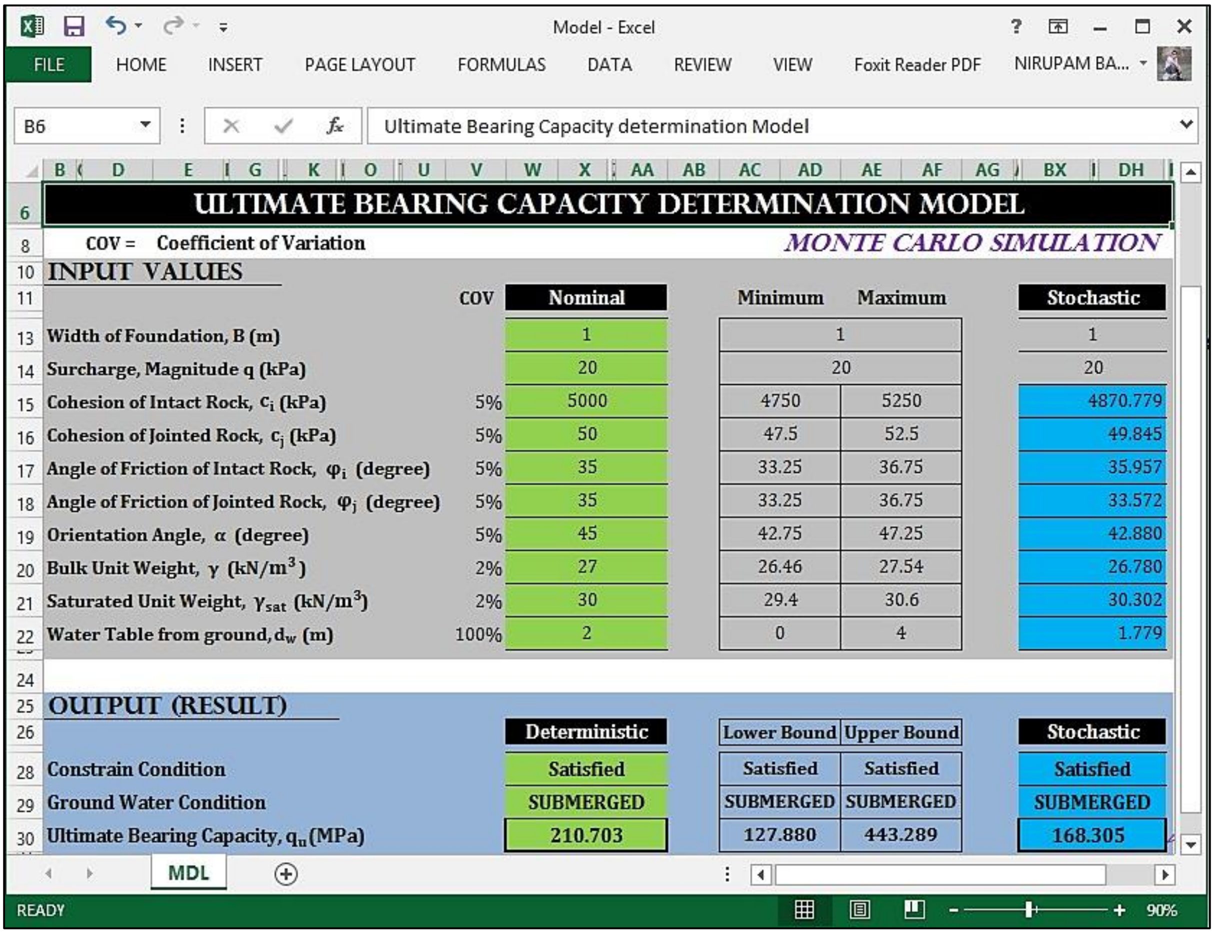
In most of the cases, rock substratum is assumed to be homogeneous, intact for simplicity in analysis. But in practical cases, rock substratum generally occurs with non-homogeneous nature with joint sets. Therefore, in-situ rock mass variability provides that deterministic analysis to be inefficient. Reliability analysis can be used to observe the performance and reliability of rock mechanics problem and also can be used for risk based decision making. Hence, a probabilistic or reliability analysis is needed to examine the uncertainties of rock mechanics problem. Therefore, an attempt has been made to model the uncertainties by reliability analysis. Theorem of determination of ultimate bearing capacity for submerged rock foundation consists of several complex equations, for which statistical calculations and reliability index determination are hard enough to compute. Among all the reliability method, Monte Carlo method is selected due to minimize computation effort and most prominently statistical calculations and reliability index can be directly computed in Monte Carlo method.

4.4.1 MONTE CARLO SIMULATION

Statistical calculations and reliability index determination by Monte Carlo method has been carried out in Microsoft excel spreadsheet. Excel formulae used for this simulation are enlisted in Appendix IV. Step by step procedure for this analysis developed in excel spreadsheet is described below:

Step 1: First of all a model capable of calculating deterministic output, lower and upper bound output along with stochastic output is created. Stochastic formulation has been developed similarly as mentioned in sub-section ‘Stochastic model creation’ in this chapter. Theorem equations are formulated for deterministic calculations as well as stochastic calculations to compare the outputs i.e. ultimate bearing capacities. Then theorem parameters foundation width and surcharge magnitude has been kept constant for all sets of calculations. Deterministic calculation set is carried out with nominal values used by Imani et al. (2012). Lower and Upper bound calculation sets are developed with minimum and maximum values for each parameter by formulating different coefficient of variation to each parameter. Thereafter with the help of random function pre-assigned in excel, stochastic calculation set is carried out with normally distributed random values ranging from minimum to maximum as formulated for lower and upper bound calculation respectively. It should be noted that every single run or trail will generate random input values within the specified range and evaluates output within the range of lower and upper bound outputs. Additionally, constrain condition and groundwater condition are also formulated to provide the information about whether the calculated ultimate bearing capacity satisfies the constrain condition or not and it is under dry or submerged condition. Cells formulated for deterministic approach are filled with green color, whereas for stochastic approach cells are filled with blue color for comparative look. A view of excel spreadsheet containing stochastic model for Monte Carlo Simulation and dataset used for the same is represented in fig. 4.21.

Step 2: Now, theorem equations are formulated again in one single row in another five spreadsheet and then 100, 500, 1000, 5000 and 10000 arrays have been developed accordingly such that each array formulated with the same set of equations. That means each row will calculate different ultimate bearing capacity for randomly generated values for each parameter. It is already mentioned that re-running the simulation will generate different set of output as inputs. After completing the simulation, reliability index and probability of failure calculations are carried out. Reliability index values and probability of failure values for 25 trails of 100, 500, 1000, 5000 and 10000 samples are stored and then plotted in graph along with mean and medians of each 25 trails. Thereafter, variation of reliability index and probability of failure with foundation width and different coefficient of variation are carried out. Graphical representation and results obtained in this step are enlisted in the following subsections in this chapter.



Model - Excel

Ultimate Bearing Capacity determination Model

ULTIMATE BEARING CAPACITY DETERMINATION MODEL

COV = Coefficient of Variation — *MONTE CARLO SIMULATION*

INPUT VALUES

	COV	Nominal	Minimum	Maximum	Stochastic
Width of Foundation, B (m)		1	1		1
Surcharge, Magnitude q (kPa)		20	20		20
Cohesion of Intact Rock, c_i (kPa)	5%	5000	4750	5250	4870.779
Cohesion of Jointed Rock, c_j (kPa)	5%	50	47.5	52.5	49.845
Angle of Friction of Intact Rock, φ_i (degree)	5%	35	33.25	36.75	35.957
Angle of Friction of Jointed Rock, φ_j (degree)	5%	35	33.25	36.75	33.572
Orientation Angle, α (degree)	5%	45	42.75	47.25	42.880
Bulk Unit Weight, γ (kN/m^3)	2%	27	26.46	27.54	26.780
Saturated Unit Weight, γ_{sat} (kN/m^3)	2%	30	29.4	30.6	30.302
Water Table from ground, d_w (m)	100%	2	0	4	1.779

OUTPUT (RESULT)

	Deterministic	Lower Bound	Upper Bound	Stochastic
Constrain Condition	Satisfied	Satisfied	Satisfied	Satisfied
Ground Water Condition	SUBMERGED	SUBMERGED	SUBMERGED	SUBMERGED
Ultimate Bearing Capacity, q_u (MPa)	210.703	127.880	443.289	168.305

Fig. 4.21: Excel spreadsheet formulated for Monte Carlo Simulation.

Step 3: From output column, i.e. column for ultimate bearing capacity, minimum and maximum bearing capacity are evaluated. For creating a histogram chart, an array of bins is developed within a range of minimum to maximum bearing capacity with 40 evenly spaced numbers. Next, an array of count is developed with the help of frequency function. This array represents the number of counts for each corresponding bin. Now, an array for scaled histogram is created to represent probability distribution such that area under the curve is equal to 1. Lastly, another array is added to represent cumulative probability with respect to bins.

Histogram chart provides a visual representation, for which it is very essential to plot histogram chart throughout statistical analysis. To create a histogram chart, first of all a 'column chart' has been created with the arrays of bins and counts. Then selected the chart area which will active two new tabs in Menu bar under 'Chart Tools', then clicked 'Select Data' nested under 'Design' tab, due to that, a new window 'Select Data Source' will be popped up. Now 'Legend Entries' are edited by putting count array in 'Series 1' and cumulative probability array in 'Series 2' and 'Horizontal Axis' is edited by putting bins array values. Again selected the chart area and clicked 'Change Chart Type' nested under 'Design' tab, due to that, a new window 'Change Chart Type' will be popped up, then clicked 'Combo' tab and then selected 'Clustered Column – Line on Secondary Axis'. With this, the critical part of plotting Histogram chart is ended. Now the columns are filled with 'violet' color and cumulative probability line is filled with 'green' color and accordingly chart area and axis labels are also modified for better visualization. Furthermore to compare histogram with probability distribution, a scaled histogram is developed with bins as horizontal axis and frequency as vertical axis.

Step 4: From the output array of ultimate bearing capacity, various statistical functions are calculated e.g. for central tendency - mean and median; for spread - standard deviation, minimum, maximum, quartiles and ranges; for shape - skewness, kurtsis. Statistical formulae used in this step are already discussed in subsection 'Statistical terms' in chapter 3. Then percentiles for 90% interval and 95% interval are also formulated. Results obtained in statistical analysis are enlisted in the next subsection 'Result Summary' in this chapter.

Step 5: For calculating confidence interval, firstly, standard error is calculated and then upper and lower confidence limits are calculated with normal distribution function. Normal distribution functions determines inverse of the standard normal cumulative distribution. The distribution has a mean of zero and a standard deviation of one.

Various analysis starting with variation of counts and cumulative probability with bins due to different sample size, variation of reliability index and probability of failure with sample size, statistical analysis e.g. central tendency, spread, shape and many others are carried out.

4.4.1.1 Effect of Sample Size

Regarding Monte Carlo simulation, it is very important to study the variation behavior of output i.e. ultimate bearing capacity due to the sample size. Therefore a study for variation behavior with increment of sample size is carried out.

4.4.1.1.1 Analysis Process

As mentioned in step 3, subsection 'Monte Carlo Simulation' different arrays comprising of bins, counts, frequency, cumulative probability are developed. Thereafter histogram charts along with scaled histograms with 40 equally spaced bins are developed for different sample size. 100, 500, 1000, 5000 and 10000 samples are used for this analysis. Dataset used for analysis for Monte Carlo Simulation is given in table 4.3.

Table 4.3: Data set used for Monte Carlo Simulation

Parameters	Coefficient of variation	Deter-ministic values	Lower Bound values	Upper Bound values
Width of Foundation, (m)	0 %	1		
Surcharge magnitude (kPa)	0 %	20		
Cohesion of Intact Rock (kPa)	5 %	5000	4750	5250
Cohesion of Jointed Rock (kPa)	5 %	50	47.5	52.5
Frictional angle of Intact Rock (degree)	5 %	35	33.25	36.75
Frictional angle of Jointed Rock (degree)	5 %	35	33.25	36.75
Orientation Angle (degree)	5 %	45	42.75	47.75
Bulk Unit Weight (kN/m^3)	2 %	27	26.46	27.54
Saturated Unit Weight (kN/m^3)	2 %	30	29.4	30.6
Water Table from ground (m)	100%	2	0	4

4.4.1.1.2: Graphical representation

Histogram charts along with scaled histograms developed for 100, 500, 1000, 5000 and 10000 samples size are represented from fig. 22 to fig. 31.

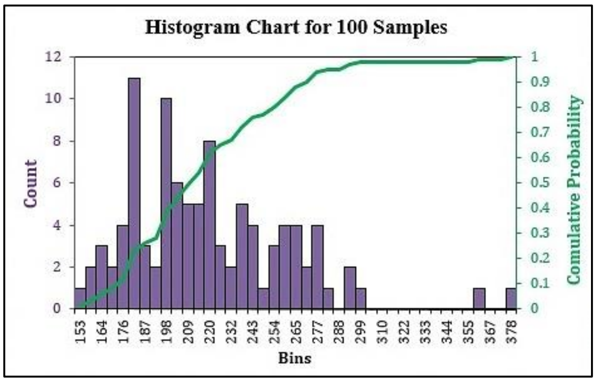


Fig. 4.22: Histogram of 100 samples

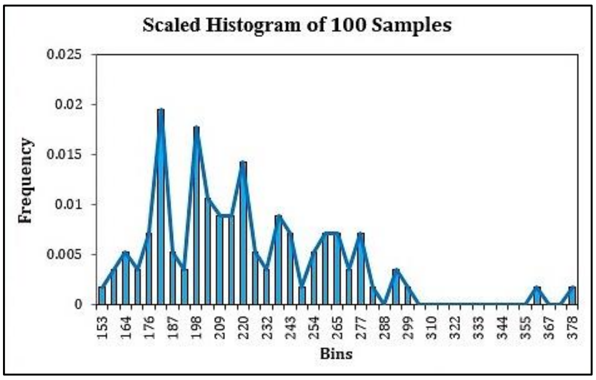


Fig. 4.23: Scaled Histogram of 100 samples

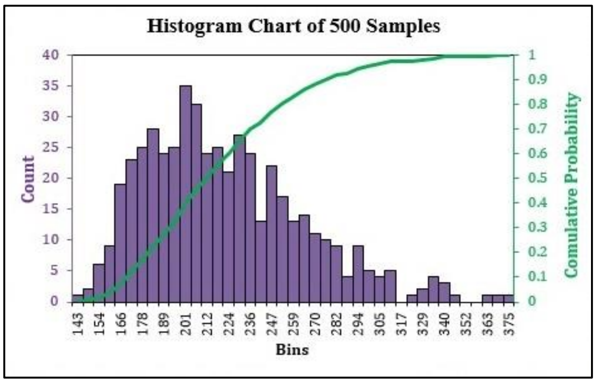


Fig. 4.24: Histogram of 500 samples

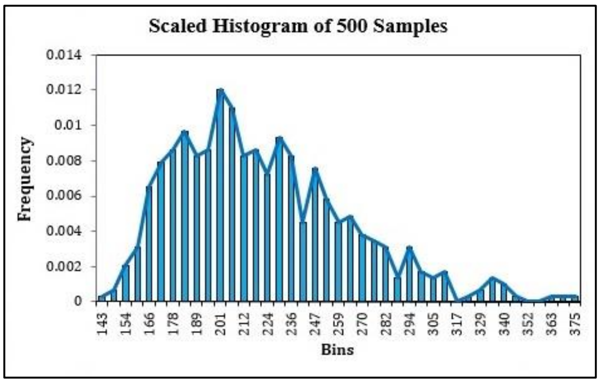


Fig. 4.25: Scaled Histogram of 500 samples

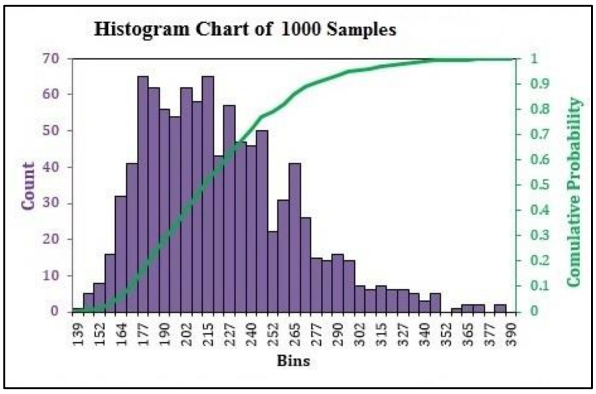


Fig. 4.26: Histogram of 1000 samples

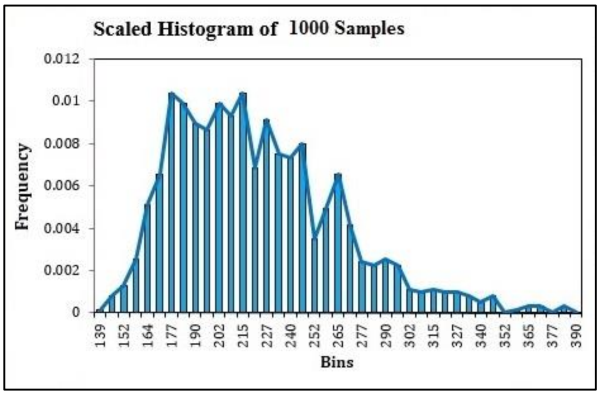


Fig. 4.27: Scaled Histogram of 1000 samples

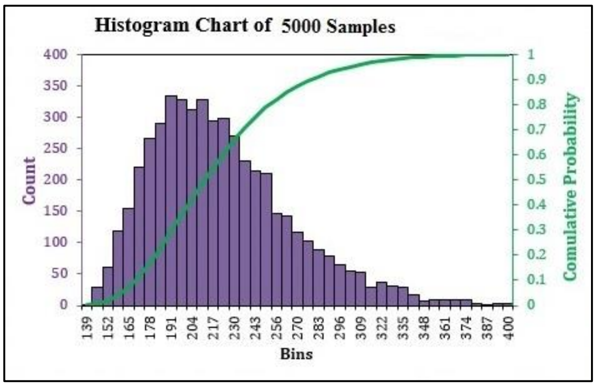


Fig. 4.28: Histogram of 5000 samples

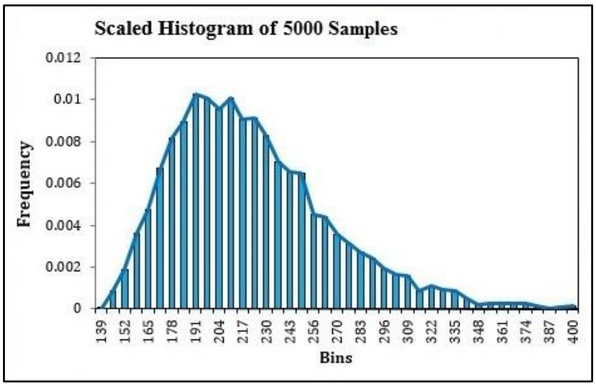


Fig. 4.29: Scaled Histogram of 5000 samples

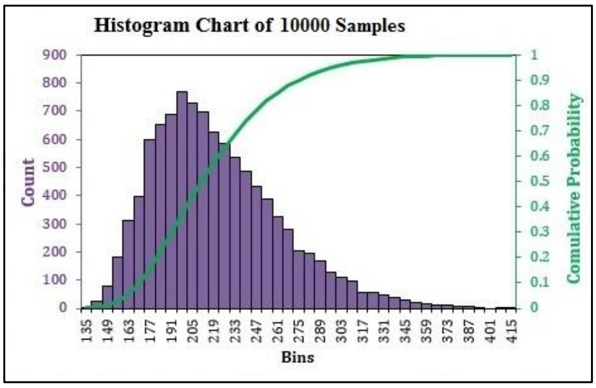


Fig. 4.30: Histogram of 10000 samples

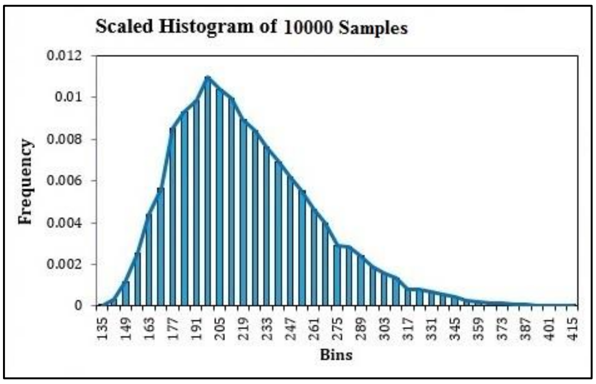


Fig. 4.31: Scaled Histogram of 10000 samples

4.4.1.1.3 Observation

From fig. 22 to fig. 31, it is clearly observed that small sample size gives totally uneven distributions for counts, frequency as well as cumulative probability. As the sample size increases distributions for counts, frequency and cumulative probability change from fairly disturbed nature to evenly distributed nature. Fig. 28 to fig. 31 represents a fair variation of counts and probability distributions but fig. 30 and fig. 31 represented for 10000 samples shows quite better distribution behavior than fig. 28 and fig. 29.

Therefore it is concluded that a higher sample size should be used for Monte Carlo Simulation such as 5000 or 10000 if possible. And hence 10000 samples are used for reliability index determination, probability of failure determination along with statistical analysis.

4.4.1.2 Determination of Reliability Index

Reliability index, β can be directly computed from 10000 sample size although it is important to study the variation behavior of reliability index with variation of sample size. Therefore to study this behavior following study has been carried out.

4.4.1.2.1 Analysis Process

Firstly reliability index is calculated for 100, 500, 1000, 5000 and 10000 samples. Thereafter, reliability index values obtained from 25 trails of 100, 500, 1000, 5000 and 10000 samples are stored. From stored data, minimum and maximum values of 25 trails for each sample size are calculated and then variation range is formulated to observe variation behavior due to sample size. To observe central tendency behavior mean and median of 25 trails for each sample size are also formulated. Table 4.4 presents the data set obtained from above formulations.

Table 4.4: Data set used for Reliability Index (β) determination

Sample Number	Minimum (β)	Maximum (β)	Variation Range	Median (β)	Mean (β)
100	4.637	5.912	1.275	5.273	5.224
500	4.775	5.618	0.842	5.122	5.117
1000	4.937	5.317	0.380	5.140	5.130
5000	4.996	5.209	0.214	5.126	5.120
10000	5.064	5.181	0.117	5.130	5.131

4.4.1.2.2 Graphical Representation

A scatter graph is plotted with every values obtained from 25 trails for every sample size as points and mean and median values are plotted as lines accordingly. To observe more precisely, points are marked in blue color whereas lines for mean and median values are plotted in red and green colors respectively.

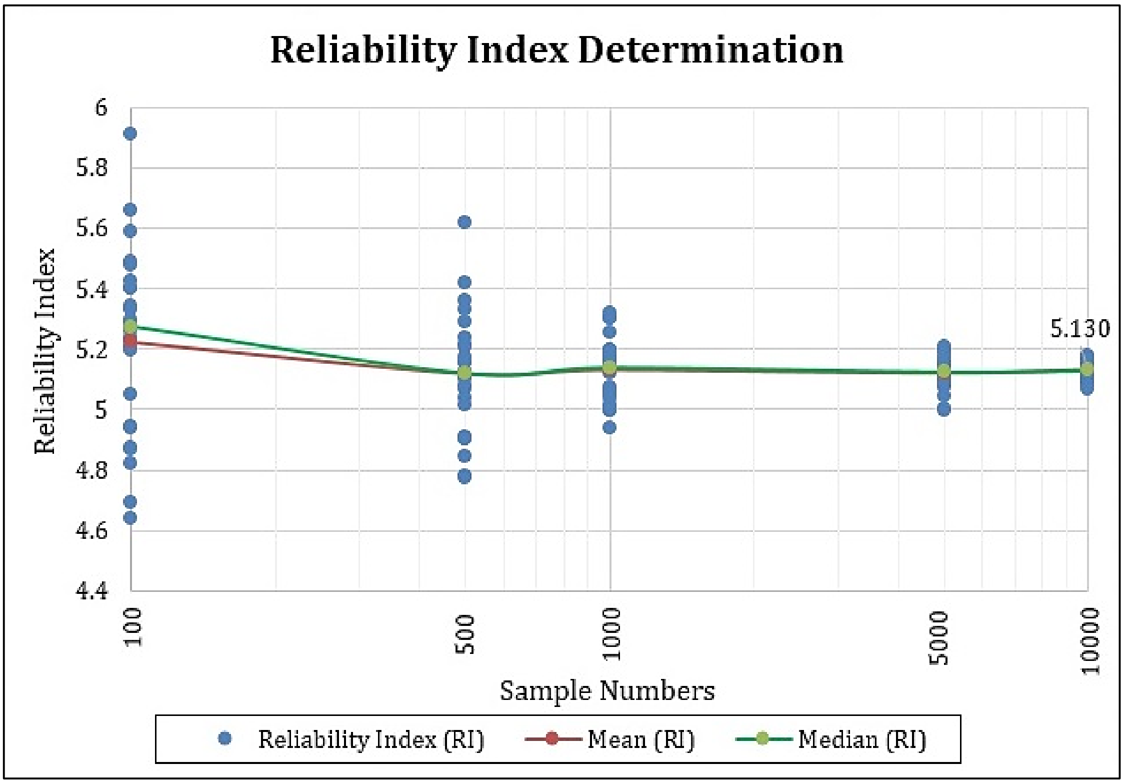


Fig. 4.32: Reliability index with variation of sample size

4.4.1.2.3 Observation

From fig. 4.32 it is observed that reliability index, β varied within a high range for sample size 100. Furthermore, central tendency behavior for sample size 100 is also poor as mean and median points are distinctly different. As the sample size increases variation range of reliability index decreases. Mean and median values are approximately same when sample size are equal or greater than 500. Most important point is, in case of sample size 10000, variation occurrence from mean or median is less than 0.1. Hence, reliability index is computed for 10000 sample size. From table 4.4, it is observed that reliability index varies from 5.064 to 5.181 and mean and median values are 5.131 and 5.130 respectively. Although mean and median both represents central tendency, but for present case, median is more valuable than mean as variation distribution is not studied. Hence, reliability index is computed to be the same with median value of 25 trails for 10000 sample size i.e. 5.130.

Therefore, it is concluded that variation of reliability index decreases with increase of sample size and hence higher sample size e.g. 10000 is better enough to compute reliability index. Another conclusion is - reliability index, β is determined as 5.130.

4.4.1.3 Determination of Probability of failure

Probability of failure, P_f can be directly computed from 10000 sample size although it is important to study the variation behavior of probability of failure with variation of sample size. Therefore to study this behavior following study has been carried out.

4.4.1.3.1 Analysis Process

Firstly probability of failure is calculated for 100, 500, 1000, 5000 and 10000 samples. Thereafter, probability of failure values obtained from 25 trails of 100, 500, 1000, 5000 and 10000 samples are stored. From stored data, minimum and maximum values of 25 trails for each sample size are calculated and then variation range is formulated to observe variation behavior due to sample size. To observe central tendency behavior mean and median of 25 trails for each sample size are also formulated. Table 4.5 presents the data set obtained from above formulations.

Table 4.5: Data set used for Probability of Failure (P_f) determination

Sample Number	Minimum (P_f)	Maximum (P_f)	Variation Range	Median (P_f)	Mean (P_f)
100	0.028	0.044	0.017	0.035	0.036
500	0.031	0.042	0.011	0.037	0.037
1000	0.034	0.039	0.005	0.037	0.037
5000	0.036	0.039	0.003	0.037	0.037
10000	0.036	0.038	0.002	0.037	0.037

4.4.1.3.2 Graphical Representation

A scatter graph is plotted with every values obtained from 25 trails for every sample size as points and mean and median values are plotted as lines accordingly. To observe more precisely, points are marked in blue color whereas lines for mean and median values are plotted in red and green colors respectively.

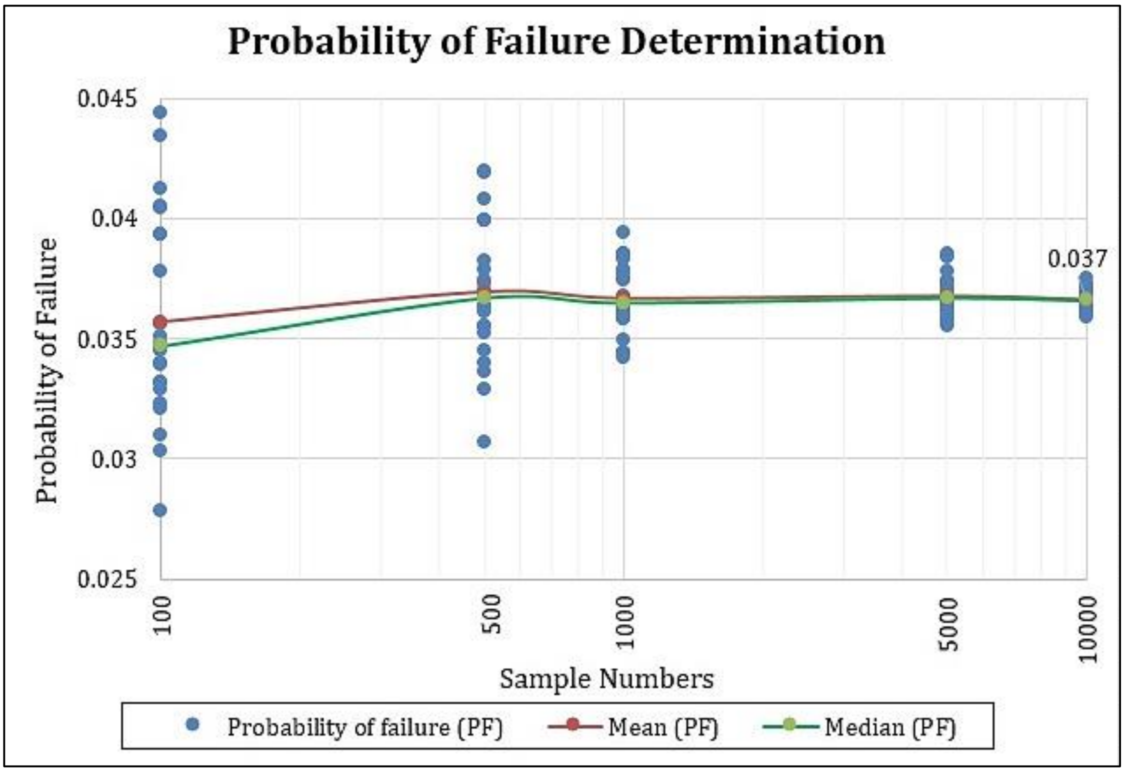


Fig. 4.33: Probability of Failure with variation of sample size

4.4.1.3.3 Observation

From fig. 4.33 it is observed that probability of failure varied within a high range for sample size 100. Furthermore, central tendency behavior for sample size 100 is also poor as mean and median points are distinctly different. As the sample size increases variation range of probability of failure decreases. Mean and median values are approximately same when sample size are equal or greater than 5000. In the case of 10000 sample size, variation occurrence from mean or median is less than 0.001. Hence, probability of failure is computed for 10000 sample size. From table 4.5, it is observed that probability of failure varies from 0.036 to 0.038 and mean and median values are 0.037 and 0.037 respectively. As the mean and median values are exactly same, hence probability of failure is computed to be the same with either values of mean and median of 25 trails for 10000 sample size i.e. 0.037.

Therefore, it is concluded that variation of probability of failure decreases with increase of sample size and hence higher sample size e.g. 10000 is better enough to compute probability of failure. Another conclusion is - probability of failure is determined as 0.037.

4.4.1.4 Result Summary

Statistical calculations and Reliability calculations through Monte Carlo simulations for 10000 samples are enlisted in table 4.8. It is already mentioned that Monte Carlo Simulation is developed in stochastic model, therefore every single simulation will generate different results.

Table 4.6: Various Results of Monte Carlo Simulation

<table>
<tr><th>Analysis</th><th colspan="2">Parameters / Terms</th><th>Value</th></tr>
<tr><td rowspan="2">Central Tendency (location)</td><td colspan="2">Mean</td><td>217.835</td></tr>
<tr><td colspan="2">Median</td><td>210.758</td></tr>
<tr><td rowspan="7">Spread</td><td colspan="2">Standard Deviation</td><td>41.92</td></tr>
<tr><td colspan="2">Minimum</td><td>135.13</td></tr>
<tr><td colspan="2">Maximum</td><td>415.36</td></tr>
<tr><td colspan="2">Range</td><td>280.23</td></tr>
<tr><td colspan="2">Quartile 3</td><td>242.33</td></tr>
<tr><td colspan="2">Quartile 1</td><td>186.73</td></tr>
<tr><td colspan="2">Interquartile Range</td><td>55.61</td></tr>
<tr><td rowspan="2">Shape</td><td colspan="2">Skewness</td><td>0.830</td></tr>
<tr><td colspan="2">Kurtsis</td><td>0.676</td></tr>
<tr><td rowspan="7">Quantiles, Percentiles,</td><td rowspan="2">90 % interval</td><td>Q (0.05)</td><td>161.337</td></tr>
<tr><td>Q (0.95)</td><td>297.235</td></tr>
<tr><td rowspan="2">95 % interval</td><td>Q (0.025)</td><td>155.205</td></tr>
<tr><td>Q (0.975)</td><td>317.340</td></tr>
<tr><td rowspan="2">Alpha, $\alpha = 0.05$</td><td>Q (α / 2)</td><td>155.205</td></tr>
<tr><td>Q (1 - α / 2)</td><td>317.340</td></tr>
<tr><td colspan="2">Standard Error</td><td>0.419</td></tr>
<tr><td rowspan="2">Reliability Analysis</td><td colspan="2">Reliability Index</td><td>5.130</td></tr>
<tr><td colspan="2">Probability of failure</td><td>0.037</td></tr>
</table>

CHAPTER 5

CONCLUSION

&

FUTURE SCOPE

5. CONCLUSION AND FUTURE SCOPE

5.1 CONCLUSIONS

On the basis of present study the following conclusions are drawn-

1. For reduction of water table depth, keeping other factors constant, ultimate bearing capacity of rock mass will be constantly increased up to the critical point and after that water table will not have any effect on ultimate bearing capacity. Highest ultimate bearing capacity occurs in critical point from where further reduction of water table will not have any effect on it. Lowest ultimate bearing capacity occurs when water table is located at ground level.
2. Ultimate bearing capacities for theorem mechanism two-sided and one-sided are exactly same for joint orientation angle 45^0. That is ultimate bearing capacity for submerged rock foundation can be determined by considering only one sided failure mechanism independently. Theorem assumption made for joint orientation angle for selection of mechanism can be eliminated.
3. As the sample size increases distributions for counts, frequency and cumulative probability change from fairly disturbed nature to stable nature. Therefore, it is recommended to use higher sample size as possible e.g. 10000.
4. Variation of reliability index decreases with increase of sample size and hence higher sample size e.g. 10000 is recommended to compute reliability index. For present case, reliability index, β is determined as 5.130.
5. Variation of probability of failure decreases with increase of sample size and hence higher sample size e.g. 10000 is recommended to compute probability of failure. For present case, probability of failure is determined as 0.037.

5.2 FUTURE SCOPES

On the basis of the present work it is observed that in the area of ultimate bearing capacity determination of rock mass following studies are essential in future –

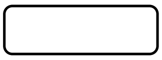
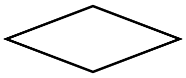
1. Theorem for determination of ultimate bearing capacity of submerged rock whose strength may be described by non-linear failure criteria to be developed.

2. In the present study, reliability analysis is based on normally distributed variable. Comparative study to be conducted with log-normal distributed or beta distributed variables.

APPENDIX - I

FLOWCHART SYMBOLS

Flowchart symbols used in the stated flowchart (Fig. 4.1) only are given below:

Flowchart Symbol	Name	Description
	Process	An operation or action step.
	Terminator	A start or stop point in a process.
	Decision	A question or branch in the process
	Data (I/O)	Indicates data inputs and outputs to and from a process
	Flow Line	Indicates the direction of flow for materials and/or information.

APPENDIX - II

MATLAB COMMANDS

MATLAB characters, commands implemented in the presented algorithm program only are given below:

Arithmetic Operators

Character	Description
+	Addition
-	Subtraction
*	Scalar and array Multiplication
/	Right division
=	Assignment operator
( )	Parentheses

Relational and Logical operators

Character	Description
<=	Less than or equal
&	Logic AND (if both are true, result is true)

Order of precedence of Arithmetic, Relational and Logical operators

Precedence Order	Operation
1	Parentheses
2	Exponentiation
3	Logical NOT
4	Multiplication, Division
5	Addition, Subtraction
6	Rational Operators
7	Logical AND

N.B. – 1. For nested parentheses, inner one will execute first than the others.

2. For operations having same precedence, expression will execute from left to right.

MATLAB character

Character	Description
'	Single quote (creates string)
;	Semicolon (suppresses display of output)
,	Comma (separates array subscript, commands)
…	Ellipsis (continuation of line)
%	Percent (suppresses display of line)

Display formats and Managing commands

Command	Description
format compact	Eliminates empty lines
clc	Clears Command Window
clear	Removes all variables from memory

Trigonometric functions

Command	Description
cosd (x)	Cosine of x (x in degrees)
sind (x)	Sine of x (x in degrees)
tand (x)	Tangent of x (x in degrees)

Input and Output commands

Command	Description
disp	Displays output directly
fprintf	Displays output in a specified format
input	Prompts for input to assign variable
\n	Starts a new line
%x.y f	Creation of fixed point notation after x spacing with y decimal points.

Flow control commands

Command	Description
case	Conditionally execute specified commands
else	Conditionally execute specified commands
elseif	Conditionally execute specified commands
end	Terminates conditional statements and loops
if	Conditionally execute specified commands
switch	Switches among several cases based on expression

APPENDIX - III

MATLAB PROGRAMS

MATLAB programs implemented in the stated algorithm program only are given below:

I. if-elseif-else-end structure

As the program executes, it reaches the *if* statement.

i. If the conditional expression is true, the program executes group 1 of commands between the *if* and the *elseif* statements and then skips to the *end*.

ii. If the conditional expression in the *if* statement is false, the program skips to the *elseif* statement. If the conditional expression in the *elseif* statement is true, the program executes group 2 of commands between the *elseif* and the *else* and then skips to the *end*.

iii. If the conditional expression in the *elseif* statement is false, the program skips to the *else* and executes group 3 of commands between the *else* and the *end*.

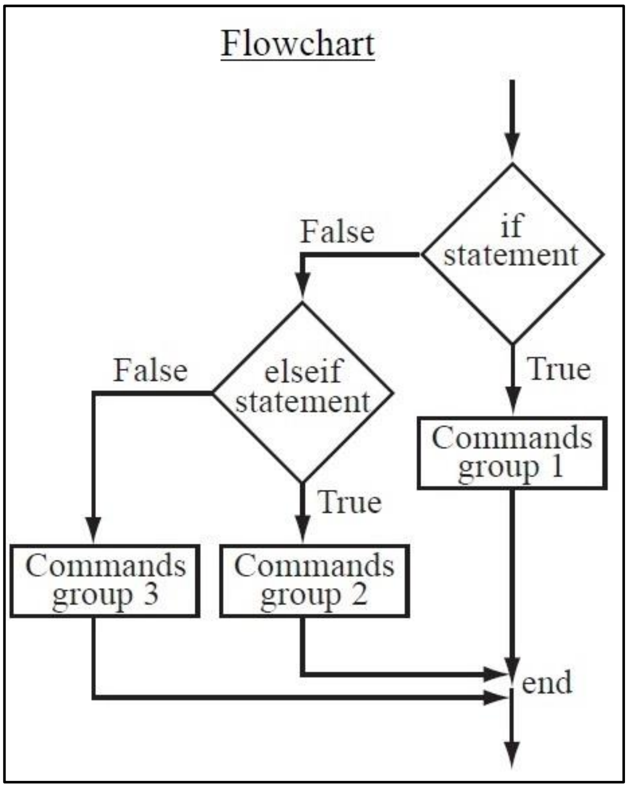


Fig.: Flow chart of if-elseif-end structure

II. switch-case statement

The first line is the switch command in the form of '*switch* switch expression'. The *switch* command is followed by one or several *case* commands. Each has a value (can be a scalar or a string) next to it (value1, value2, etc.) and an associated group of commands below it. After the last case command there is an optional otherwise command followed by a group of commands. The last line must be an end statement. After execution of program if there is more than one match, only the first matching *case* is executed. If no match is found and the *otherwise* statement (which is optional) is present, the group of commands between *otherwise* and *end* is executed. If no match is found and the *otherwise* statement is not present, none of the command groups is executed.

```
......        MATLAB program.
......

switch  switch expression
     case  value1
     ........
     ........          Group 1 of commands.
     case  value2
     ........
     ........          Group 2 of commands.
     case  value3
     ........
     ........          Group 3 of commands.
     otherwise
     ........
     ........          Group 4 of commands.
end
......
......       MATLAB program.
```

Fig.: Flow chart of switch-case statement

APPENDIX - IV

MICROSOFT EXCEL FORMULAE

Microsoft excel formulae used in the study of comparison between mechanisms and reliability analysis only are given below:

Basic Math Functions

Function	Description
+	Add numbers.
-	Subtract numbers.
*	Multiply numbers.
/	Divide numbers.

Math and Trigonometry Functions

Syntax	Description
ATAN(number)	Returns the arctangent, or inverse tangent, of a number (in radian).
COS(number)	Returns the cosine of the given angle (in radian).
DEGREES(angle)	Converts radians into degrees.
EXP(<number>)	Returns e (=2.71828182845904) raised to the power of a given number.
SIN(number)	Returns the sine of the given angle (in radian).
RADIANS(angle)	Converts degrees to radians.
SQRT(number)	Returns a positive square root.
SUM(number1,[number2],...])	Adds all the numbers specified as arguments. Each argument can be a range, a cell reference, an array, a constant, a formula, or the result from another function.
TAN(number)	Returns the tangent of the given angle (in radian).

Logical Functions

Syntax	Description
IF(logical_test, [value_if_true], [value_if_false])	Returns one value if a condition you specify evaluates to TRUE, and another value if that condition evaluates to FALSE.

Statistical Functions

Syntax	Description
AVERAGE(number1,[number2], ...)	Returns the average (arithmetic mean) of the arguments.
COUNT(value1,[value2], ...)	Counts the number of cells that contain numbers, and counts numbers within the list of arguments.
FREQUENCY(data_array,bins_array)	Calculates how often values occur within a range of values, and then returns a vertical array of numbers.
MAX(number1,[number2] ,...)	Returns the largest value in a set of values.
MEDIAN(number1,[number2], ...)	Returns the median of the given numbers.
MIN(number1,[number2], ...)	Returns the smallest number in a set of values.
KURT(number1,[number2], ...)	Returns the kurtosis of a data set. Kurtosis characterizes the relative peakedness or flatness of a distribution compared with the normal distribution.
SKEW(number1,[number2], ...)	Returns the skewness of a distribution. Skewness characterizes the degree of asymmetry of a distribution around its mean.

Compatibility Functions

Syntax	Description
NORMSINV(probability)	Returns the inverse of the standard normal cumulative distribution. The distribution has a mean of zero and a standard deviation of one.
PERCENTILE(array,k)	Returns the k-th percentile of values in a range.
QUARTILE (array,quart)	Returns the quartile of a data set.
STDEV(number1,[number2],...])	Estimates standard deviation based on a sample.

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